Optimization Model for Disc Cam Flat Faced Follower Mechanism

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Abstract Multiobjective optimization has become a necessity since optimization of one parameter may increase another, which may lead to increase in overall increase of cost. Thus, the aim is to reduce overall cost by reducing material, maintenance and performance cost of a device. The paper aims to analyse the mechanism of flat faced follower combined with a Disc Cam and to formulate a multiobjective optimization model of the mechanism. The geometric parameters, aimed to be optimised, include cam base circle radius, cam disc width and eccentricity of follower translating axis. The performance parameters are torque required to rotate the cam and the contact stresses developed at the cam follower contact surface which are related to performance cost and maintenance cost. The model is subjected to constraints regarding the maximum contact stress, radius of curvature, and a finite force required to lift the follower. Further the optimization parameters are bounded by the upper and lower values. The variation of the parameters with cam rotation angle has been displayed using MATLAB and conclusions are drawn.

Keywords Multiobjective optimization, cam mechanism, maximum contact stress, flat faced follower, MATLAB.

I. INTRODUCTION

The cam is a mechanical member used to impart desired motion to a follower by direct contact. In general, the flat faced follower combined with a disc cam (Fig 1) is used to work under high speed, light load and in limited space. But as size of the device reduces there is an increase in contact stresses. Further the mechanism has to work without being jammed in the guides and follower must follow the desired path. Base circle radius of the cam is the least radius of the cam. The type of motion and stroke of the follower decides the cam profile radius of curvature. The contact of cam and follower theoretically must be a line contact, but practically the contact is a surface contact.

A static analysis reveals the primary forces acting on the mechanism and various parameters on which the forces depend.

II. LITERATURE REVIEW

Researches have been done and papers published on various aspects of design and analysis of cam follower mechanism. Cam and follower design optimization is solely based on the level of analysis of the designer. Increasing the number of relational constraints on the objective parameters increases the practical accuracy of the model. Flores, P. (2013) [1] presented a computational approach for design optimization of cam mechanism with eccentric translating roller follower. A non-linear multifunction decision model was formed with cam base circle radius, rise pressure angle and return pressure angle as the objective parameters. Tsiafis, I. et al. (2013) [2] worked on solving an optimal model of a cam mechanism with translating flat face follower using genetic algorithm. Moise, V. et al. (2011) [3] evaluated the relation between cam size optimization and the radius of curvature of the cam profile. The minimum dimension of disc cam with translating flat face follower was determined, imposing constraint that the radius of curvature of the active surface of the cam doesn’t change the sign, in terms of design requirements. Mali, M. R. et al. [2012] [4] worked to change the flat face follower to a curved face follower, so that line contact can be changed to point contact. The finite element approach was used to perform the analysis. The results indicated change of flat face of a roller follower to a curved face roller mechanism. This was seen to lower frictional losses which results in improved mechanical efficiency.

III. OBJECTIVE

The objectives of this study are to investigate the various follower and disc cam for better
understanding of the mechanism, form multiobjective optimization functions with constraints to restrict the practical unfeasibility through detailed analysis. Lastly, analyse the variation of the parameters as function of cam angle.

**IV. METHODOLOGY**

A detailed study was done on the cam mechanism that showed that roller and flat faced follower are primarily used. Knife edge and spherical find no practical use. The flat faced can be designed for compact use because of its simplicity and decrease in jamming problem with reduction in cam size. It was also found that pressure angle is always zero for such arrangement and so it doesn’t play a role in designing.

A static analysis was done on a basic arrangement to find the relation of the various parameters with jamming of follower in the guides. Constraints regarding radius of curvature, offset, length of follower face, offset, and maximum contact stress were noted. Various interrelations of the parameters were found and relations of the parameters with cam angle rotation were derived. Graphs were prepared through MATLAB coding to check the variation of the important parameters with cam angle rotation.

**V. ANALYSIS & DISCUSSION**

**A. Study of cam mechanism**

A study of the optimization of cam mechanism is beneficial in understanding the factors that affect the design parameters and the limitation imposed on optimization. The study was restricted to disc type cam and follower arrangement.

The knife edge follower is the simplest of all but rarely used as the contact stresses are extremely high, which leads to excessive wear. The roller and flat faced follower find practical use. In case of steep rise, roller follower has a tendency to jam in the guides due to high side thrust. The flat faced follower has similar tendency but the cause is different. For flat faced follower, contact between cam and follower occurs along the face of the follower. Due to the eccentricity of the contact, with the follower axis, the follower has a tendency to overturn in its guide and jam. For a roller follower the solution is to increase the size of cam and for a flat faced follower the solution is to decrease the size. In general, for such reasons, the flat faced follower is used where the space is limited. The flat faced follower is commonly used for higher speeds and lighter loads. It can handle lighter loads as the friction increases with increase in weight. On the contrary it has the benefit of higher speed as it is simple in construction unlike roller follower.

A disc cam and flat faced follower is taken under investigation for the paper. The optimization model is aimed to minimize the size of the mechanism to produce a compact device, the torque required to drive the device and the contact stresses.

**B. Minimization functions and objective parameters**

The primary forces (Fig 2) seen in the mechanism are force required to lift the follower (F₁), force on follower due to inertial effect, weight of follower and spring compression (P), frictional forces between the guide of the follower and the follower (F₁ and F₂), frictional force at the cam and follower contact surface (F), normal forces at guide (N₁ and N₂).

[Fig. 2 A disc cam and follower arrangement with with various forces.]

[Fig 3 Parameters and their explanation]

The base circle radius (R₀), width of cam(L) and the offset (e) (Fig 3) taken are taken as the geometric parameters in the model that affect the size of mechanism. Although length of follower face (Lₐ) (Fig 3) is a primary geometric parameter, it cannot be optimized as it depends upon the stroke which has a fixed value [5] as specified for a particular purpose. Parameters like diameter of guides (d), distance from cam follower contact to guide (b), and length of guide (lₚ) (Fig 3) are not considered as they are mechanism arrangement dependent.

From the performance point of view, the torque requirement and maximum contact stress are included in the functions. Torque (T) requirement must be low so that mechanism needs less energy to
run. A low contact stress ($\sigma_{\text{max}}$) will lower the maintenance cost due to less wear and materials having low permissible strength can be used.

The minimization functions are:

Min $Z_1 = R_b + L + e$

Min $Z_2 = T$

Min $Z_3 = \sigma_{\text{max}}$

C. Constraints

A static analysis of the system reveals the interrelation of the forces and design parameters. For symbols refer TABLE 2.

\[ F = \mu_2 F_c \]

\[ N_1 = \frac{F_c - P}{2\mu_1} - \frac{F_c \mu_2}{2} \]

\[ N_2 = \frac{F_c - P}{2\mu_1} + \mu_2 F_c \]

\[ F_c = \frac{1}{2\mu_1} - \frac{\mu_2}{2} + \mu_2 \times b - (\alpha - e) \]

\[ F_i = \mu_1 \times N_1 \]

\[ F_2 = \mu_2 \times N_2 \]

For the follower to ascend in the guides without jamming

\[ \log \left( \frac{1}{2\mu_1} + \frac{\mu_2}{2} \right) + \mu_2 \times b - (\alpha - e) > 0 \]

To avoid undercut and cusp

\[ \rho > 0 \]

From [5]

\[ \rho = R_b + f(\theta) + f''(\theta) \]

D. Input parameters and formula:

**TABLE 1**

<table>
<thead>
<tr>
<th>Kinematics Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of Dwell ($\beta'$)</td>
</tr>
<tr>
<td>Angle of Rise ($\beta_1$)</td>
</tr>
<tr>
<td>Angle of Dwell ($\beta''$)</td>
</tr>
<tr>
<td>Angle of Fall ($\beta_2$)</td>
</tr>
<tr>
<td>Follower Motion Rise</td>
</tr>
<tr>
<td>Follower Motion Fall</td>
</tr>
</tbody>
</table>

The material of cam follower and guide is steel.

**TABLE 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>Stroke of follower</td>
<td>0.006 m</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>Coefficient of friction between follower and guide</td>
<td>0.16</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>Coefficient of friction between follower and cam</td>
<td>0.16</td>
</tr>
<tr>
<td>$b_{\text{min}}$</td>
<td>Min distance between cam follower contact and guide</td>
<td>0.01 m</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Angle of rise</td>
<td>70°</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>Angle of return</td>
<td>80°</td>
</tr>
<tr>
<td>$I_g$</td>
<td>Length of guide</td>
<td>0.02 m</td>
</tr>
<tr>
<td>$R_b^l$</td>
<td>Base circle radius lower limit</td>
<td>0.03 m</td>
</tr>
<tr>
<td>$R_b^u$</td>
<td>Base circle radius upper limit</td>
<td>0.06 m</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>Poisson ratio follower material</td>
<td>0.3</td>
</tr>
<tr>
<td>$\nu_2$</td>
<td>Poisson ratio cam material</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma_{\text{per}}$</td>
<td>Permissible stress of cam and follower material</td>
<td>1.2 x 10^9 N/m²</td>
</tr>
<tr>
<td>$y_i$</td>
<td>Initial compression in spring</td>
<td>0.001 m</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Mass of follower</td>
<td>0.03 kg</td>
</tr>
<tr>
<td>N</td>
<td>Cam Rotation speed</td>
<td>240 rpm</td>
</tr>
<tr>
<td>$E_1$</td>
<td>Youngs modulus of follower material</td>
<td>2068427 N/m²</td>
</tr>
<tr>
<td>$E_2$</td>
<td>Youngs modulus of cam material</td>
<td>2068427 N/m²</td>
</tr>
<tr>
<td>$L_u$</td>
<td>Lower limit for cam width</td>
<td>0.08 m</td>
</tr>
</tbody>
</table>
The parameters are interrelated with cam angle as follows [5,6,7]:

\[ \sigma_{\text{max}} = \frac{2F_c}{\pi c} \]  
\[ c = \sqrt{\frac{4F_c \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}{\pi L \left( \frac{1}{\rho^*} \right)}} \]  
\[ T = \frac{F_c \times v_f}{\omega} \]  
\[ h = \frac{1-\cos 2\pi \left( \frac{\theta}{\beta_1} \right)}{\beta_1} \]  
\[ f''(\theta) = h \left( \frac{1-\cos 2\pi \left( \frac{\theta}{\beta_1} \right)}{\beta_1^2} \right) \]  
\[ \alpha_{\text{max}} = \frac{2h}{\beta_1} \]  
\[ f(\theta) = y_f \]  
\[ b = b_{\text{min}} + y_f \]  
\[ P = m_f \times a_f + k(y_f + y_i) + W_f \]  
\[ y_f = h \left( \frac{\theta}{\beta_1} - \frac{\sin 2\pi \left( \frac{\theta}{\beta_1} \right)}{2\pi} \right) \]  
\[ v_f = h \left( \frac{1-\cos 2\pi \left( \frac{\theta}{\beta_1} \right)}{T} \right) \]  

\[ a_f = h \left[ \frac{2\pi \sin 2\pi \left( \frac{\theta}{\beta_1} \right)}{T^2} \right] \]  

\[ \theta \] Cam angle 0° to 360°
\[ \alpha \] Eccentricity of contact point of cam follower
\[ y_f \] Displacement
\[ v_f \] Velocity
\[ a_f \] Acceleration
\[ f(\theta) \] Displacement in terms of \( \theta \)
\[ f''(\theta) \] 2nd derivative of \( y_f \) with respect to cam angle

\[ \sigma_{\text{max}} = \frac{2F_c}{\pi c} \]  
\[ c = \sqrt{\frac{4F_c \left( \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)}{\pi L \left( \frac{1}{\rho^*} \right)}} \]  
\[ T = \frac{F_c \times v_f}{\omega} \]  
\[ h = \frac{1-\cos 2\pi \left( \frac{\theta}{\beta_1} \right)}{\beta_1} \]  
\[ f''(\theta) = h \left( \frac{1-\cos 2\pi \left( \frac{\theta}{\beta_1} \right)}{\beta_1^2} \right) \]  
\[ \alpha_{\text{max}} = \frac{2h}{\beta_1} \]  
\[ f(\theta) = y_f \]  
\[ b = b_{\text{min}} + y_f \]  
\[ P = m_f \times a_f + k(y_f + y_i) + W_f \]  
\[ y_f = h \left( \frac{\theta}{\beta_1} - \frac{\sin 2\pi \left( \frac{\theta}{\beta_1} \right)}{2\pi} \right) \]  
\[ v_f = h \left( \frac{1-\cos 2\pi \left( \frac{\theta}{\beta_1} \right)}{T} \right) \]  

\[ a_f = h \left[ \frac{2\pi \sin 2\pi \left( \frac{\theta}{\beta_1} \right)}{T^2} \right] \]  

**E. Graphs**

A MATLAB code was written to show the variation of the parameters as a function of cam angle. The equations (1) to (22) and the data from TABLE 1, TABLE 2 were used in the code.
VI. CONCLUSION

Cam and follower mechanism is very helpful in providing complex motion, but frictional and contact stress act as limiting factors. Furthermore, flat faced follower cannot trace negative or zero radius of curvature. The cam profile can be traced using the displacement graph (Fig 4). The velocity graph (Fig 5) changes direction during fall but the peak values are same. It can be seen from the acceleration graph (Fig 6) that small angle for rise or fall will result in fast chances in acceleration which will lead to jerks. The torque versus cam angle graph (Fig 7) shows that the torque requirement is positive during the rise of follower and negative during the descending of follower. The magnitude of peak torque during ascending is more than the value during descending. The graph of frictional force versus cam angle show that the frictional forces vary, almost in the same trend. During the rise of follower, the forces increase and decrease to reach a peak value. For the dwell period the forces remain constant. The variation during rise and fall is due to the variation in normal force on the contact surface. The follower force (Fig 9) has similar changes in the graph like the frictional forces (Fig 8) but the variations are less in comparision. The radius of curvature graph (Fig 10) is sinusoidal during the the rise and fall of cam angle. The variation is steeper and the peak values are also more in the rise. The maximum contact stress graph (Fig 11) shows that the stress increases rapidly to a peak point than decreases to a constant value during dwell. During the return motion of the follower, the stress varies in similar trend but the peak value during return is much less. The optimizarion model along with the MATLAB graph helped to understand the variation of the performance parameters as a function of the cam angle. This model can be used to understand the behaviour for different geometric parameters with a similar arrangement. Thus the model will help in pre-fabrication analysis of the mechanism.
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