Secrecy Outage Analysis of Underlay Cognitive Radio Over Nakagami-Q (Hoyt) Fading Channels

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Abstract:
An underlay cognitive radio unit over the Nakagami-q fading channel which consists of a source S, a secondary user (SU) and an eavesdropper who wants to eavesdrop the information between S and SU, is studied. The broadcast power of S is simultaneously adjusted according to the channel state information of S-PU link and a given threshold interference of that primary user can permit. A closed form analytical expressions of Secrecy Outage Probability has been derived. The robustness of our analysis models are verified by simulation results.

Index terms: Secrecy Outage Probability, Nakagami-q (Hoyt), Underlay Cognitive radio networks(CRNs)

I. Introduction

Spectrum scarcity and poor spectrum utilization are two contradictory statements that makes to focus public on cognitive radio network, which makes secondary users (SUs) enable to share the frequency band of primary users(PUs). The three different models of cognitive radio networks (CRNs) enable users to share their frequency bands are underlay, overlay, interweave etc. [10]. Among the models that we have discussed the underlay type of strategy is easy to realize, as SUs are needed to just adjust the power of them within the threshold level that PUs can tolerate without experiencing a complex calculation.

The security of CRNs discussed by [2] to [11]. In [2] secrecy capacity for a multi-antenna SU transmitter in the presence of eavesdropper is studied. Reference [3] discusses the secure resource allocation in CRNs for guaranteeing a secrecy rate for PUs. A secure medium access control (MAC) is proposed in [4] for CRNs. Some secure broadcasting in non-CRNs over independent/correlated Rayleigh [5], [6] Aog-normal [7], Gaussian fading channels [8]. Reference [9], [10] studies about the Nakagami-m fading channels, which include special cases like one-sided Gaussian distribution (m=0.5), Rayleigh(m=1) but they didn't mention about the security problems in CRNs. [11] studies the secrecy outage probability (SOP) and the probability of non-zero secrecy capacity (PNSC) of underlay cognitive radio has been derived by using the closed form expressions of both SOP and PNSC. According to the best of my knowledge, SOP over Hoyt distribution has not been investigated.

In this paper, we study the SOP of the underlay cognitive radio unit as shown in fig.1 over Nakagami-q fading channel (Hoyt distribution) and the closed form expression of the Secrecy Outage Probability (SOP) is derived.

II. System Model

We consider the system model same as the [1] but the fading that channel experience is Nakagami-q fading. PU_Tx and PU_Rx are primary user’s transmitter and receiver of the underlay cognitive radio networks unit as shown in Fig.1.

![Fig. 1 System model](http://www.ijettjournal.org)
In the secondary system, a Source (S) sends confidential information to destination SU, while eavesdroppers want to eavesdrop the confidential information. All the channels \( (h_i, i \in \{1,2,3\}) \) shown in Fig. 1 are assumed to experience independent Nakagami-q fading with parameters \( m_i, \Omega_i, i \in \{1,2,3\} \) and Additive White Gaussian Noise with power density, \( N_0 \). The channel state information \( (h_i, i \in \{1,2,3\}) \) is assumed to be available at S. Though \( h_1 \) is unavailable when Eav keeps in silence and just listens, we assume \( h_1 \) is available at S to set up analysis models to study the secrecy outage performance for every realization of \( h_3 \). The peak interference power from S which PU Rx can tolerate is \( P_{th} \). \( P \) denotes the maximum transmit power at S. In the underlay scheme, the interference power received at PU Rx must be within \( P_{th} \), such that \( P = P_{th}|h_3|^2 \). In this work, we assume that SU is located far from PU Tx. Then, the received signals at SU will not be influenced by PU system. Thus, the signal to noise ratio at SU and Eav can be written as \( SU = P|h|^2/N_0 = P_{th}|h_1|^2/N_0|h_3|^2 \), \( E = P|h|^2/N_0 = P_{th}|h_1|^2/N_0|h_3|^2 \), respectively.

### III. Secrecy outage Probability Analysis

The probability that secrecy capacity is smaller than that of threshold secrecy capacity \( C_{th} \) is defined as SOP. SOP can be expressed as [12]

\[
SOP(C_{th}) = P(C_{SU} \leq C_{th})
\]

\[
= P\left( \frac{1}{2} \log_2\left( 1 + \gamma SU \right) - \frac{1}{2} \log_2\left( 1 + \gamma E \right) \leq C_{th} \right)
\]

\[
= P\left\{ \frac{1}{2} \log_2\left( 1 + \gamma SU \right) \leq C_{th} \right\}
\]

Substituting \( \gamma SU \) and \( \gamma E \) into Eq. (1), we have

\[
SOP(C_{th}) = P\left\{ 1 - \frac{\log_2\left( 1 + \gamma SU \right)}{2} \right\} = P\left\{ 1 - \frac{a}{2} \right\}
\]

\[
SOP(C_{th}) = \frac{P_{th}}{N_0} \left| h_1 \right|^2
\]

Thus, SOP can be computed as

\[
SOP(C_{th}) = \int_{0}^{\infty} pdf_z(z)dz = 1 - \int_{1}^{\infty} pdf_z(z)dz.
\]

Thus, as shown in Eq (3), to obtain the closed-form expression of SOP, we should characterize the pdf of the positive random variable, \( Z \).

The pdf of the channel power gain over Nakagami-q or Hoyt fading channel can be given by [12]

\[
f_\beta(\beta) = \frac{1 - q^2}{2q\Omega} \exp(-\frac{1 + q^2}{4q^2\Omega}) I_0\left(\frac{1 - q^4}{4q^2\Omega}\right)\beta
\]

where \( \beta = |h|^2, \Omega = \mathbb{E}[\beta] \)

Let \( a_1 = P_{th}/N_0, a_2 = \lambda - 1, a_3 = \lambda P_{th}/N_0 \). It is obvious that \( a_i > 0, i \in \{1,2,3\} \), such that

\[
Z = \frac{a_i |h_i|^2}{a_2 |h_2|^2 + a_3 |h_3|^2}
\]

The pdf of \( X = a_i |h_i|^2 \) can be computed as follows:

\[
f_x(x) = \frac{1}{a_i^2 |h_2|^2 + a_3 |h_3|^2} \exp\left(-\frac{1 + q^2}{4q^2\Omega a_i}\right) I_0\left(\frac{1 - q^4}{4q^2\Omega a_i}\right)
\]

\[
= \frac{1 + q^2}{2q\Omega a_i} \exp\left(-\frac{(1 + q^2)^2}{4q^2\Omega a_i}\right) I_0\left(\frac{(1 - q^4)}{4q^2\Omega a_i}\right)
\]

We can write the pdf of \( Z_i = a_2 |h_2|^2 + a_3 |h_3|^2 \) as

\[
f_{Z_i}(z_i) = \int_{0}^{z_i} f_{a_i|h_i|^2}(x) f_{a_i|h_i|^2}(z_i - x) dx
\]

\[
= k \int_{0}^{z_i} \exp(ax) I_0(bx) I_0(c(z_i - x)) dx
\]

where

\[
k = \frac{1 + q^2}{4q^2\Omega a_i} \frac{1 + q^2}{4q^2\Omega a_i} \exp\left(-\frac{(1 + q^2)^2}{4q^2\Omega a_i}\right)
\]

\[
a = \frac{(1 + q^2)^2}{4q^2\Omega a_i} \frac{(1 + q^2)^2}{4q^2\Omega a_i} b = \frac{1 - q^4}{4q^2\Omega a_i}
\]

\[
c = \frac{1 - q^4}{4q^2\Omega a_i}
\]

By expanding \( I_0 \) terms

\[
f_{Z_i}(z_i) = k \int_{0}^{z_i} \exp(ax) \left[ \int_{0}^{b} e^{-bx} dx \right] \left[ \int_{c}^{\infty} e^{(z_i-x)bx} dx \right] dx
\]

\[
= k \int_{0}^{z_i} \exp(ax) \left[ e^{bx} - \frac{1}{b} e^{bx} \right] \left[ e^{(z_i-x)bx} \right] dx
\]

\[
= k \int_{0}^{z_i} \exp(ax) \left[ e^{bx} - \frac{1}{b} e^{bx} \right] \left[ e^{(z_i-x)bx} \right] dx
\]
\[ f_s(z) = k \pi^2 \int_0^1 e^{\frac{a+bn}{2}(z-\frac{x}{2}) - \frac{bx}{6}} \left( \frac{1}{6} + \frac{1}{36} bx(c(z_1-x)^2) \right) dx \tag{7} \]

After Integration we get
\[
\begin{align*}
  f_s(\zeta_1) &= K \pi^2 + \frac{b}{(a+b+c)^2} \left[ \left( \frac{z_1}{6(a+b+c)^2} + \frac{1}{3(a+b+c)^2} \right) + \frac{e^{\frac{a+bn}{2}z_1} - e^{\frac{a+bn}{2}}}{\frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)}} \right] \\
  f_s(\zeta_2) &= K \pi^2 + ce^{\frac{a+bn}{2}} - be^{\frac{a+bn}{2}z_1} \left( \frac{z_1}{6(a+b+c)^2} + \frac{1}{3(a+b+c)^2} \right) + e^{\frac{a+bn}{2}z_1} + e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) \\
  f_s(\zeta_3) &= K \pi^2 + ce^{\frac{a+bn}{2}} - be^{\frac{a+bn}{2}z_1} \left( \frac{z_1}{6(a+b+c)^2} + \frac{1}{3(a+b+c)^2} \right) + e^{\frac{a+bn}{2}z_1} + e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) \\
  f_s(Z) &= K \pi^2 + ce^{\frac{a+bn}{2}} - be^{\frac{a+bn}{2}z_1} \left( \frac{z_1}{6(a+b+c)^2} + \frac{1}{3(a+b+c)^2} \right) + e^{\frac{a+bn}{2}z_1} + e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) \\
  f_s(Z) &= K \pi^2 + ce^{\frac{a+bn}{2}} - be^{\frac{a+bn}{2}z_1} \left( \frac{z_1}{6(a+b+c)^2} + \frac{1}{3(a+b+c)^2} \right) + e^{\frac{a+bn}{2}z_1} + e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) \\
  \int_{0}^{\infty} f_s(Z) dz &= \int_{0}^{\infty} e^{\frac{a+bn}{2}z_1} - e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) dz \\
  \int_{0}^{\infty} f_s(Z) dz &= \int_{0}^{\infty} e^{\frac{a+bn}{2}z_1} - e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) dz \\
  SOP(C_{ab}) &= 1 - \int_{0}^{\infty} f_s(Z) dz \\
  SOP(C_{ab}) &= 1 - \int_{0}^{\infty} e^{\frac{a+bn}{2}z_1} - e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) dz \\
  SOP(C_{ab}) &= 1 - \int_{0}^{\infty} e^{\frac{a+bn}{2}z_1} - e^{\frac{a+bn}{2}} \left( \frac{1}{6} - \frac{2bc \pi^2}{36(a+b+c)^2} \right) dz \\
  \frac{1+q_i^2}{2q_i \Omega_i a_i} - \exp \left( \frac{(1+q_i^2)^2 z}{4q_i^3 \Omega_i a_i} \right) I_0 \left( \frac{(1-q_i^2)^2 z}{4q_i^3 \Omega_i a_i} \right) dy \
  SOP(C_{ab}) &= 1 - \frac{(\Omega_i a_i)^2}{(\Omega_i a_i + \Omega_j a_j)(\Omega_i a_i + \Omega_j a_j)} \tag{14} \\
  IV. Results and Discussions

For q_1 = q_2 = q_3 = 1 Equation (5) becomes
\[ f_s(x) = \frac{1}{\Omega_i a_i} \exp(-\frac{x}{\Omega_i a_i}) I_0(0) \tag{9} \]
\[ f_s(x) = \frac{1}{\Omega_i a_i} \exp(-\frac{x}{\Omega_i a_i}) \tag{9a} \]
\[ f_s(Z) = \frac{1}{\Omega_i a_i} \int_{0}^{\infty} \exp(-\frac{x}{\Omega_i a_i}) \frac{1}{\Omega_i a_i} \exp(-z_1 x) \Omega_i a_i dx \tag{10} \]
\[ f_s(z) = \frac{1}{\Omega_i a_i} \left( \exp(-\frac{z_1 x}{\Omega_i a_i}) - \exp(-\frac{z_1 x}{\Omega_i a_i}) \right) \tag{10a} \]
\[ f_s(z) = \frac{y}{\Omega_i a_i - \Omega_j a_j} \left( e^{-\frac{z_1 x}{\Omega_i a_i}} - e^{-\frac{z_1 x}{\Omega_j a_j}} \right) \tag{11} \]
\[ f_s(z) = \frac{1}{\Omega_i a_i - \Omega_j a_j} \left( \frac{1}{\Omega_i a_i - \Omega_j a_j} \right) \tag{12} \]
\[ SOP(C_{ab}) = 1 - \frac{(\Omega_i a_i)^2}{(\Omega_i a_i + \Omega_j a_j)(\Omega_i a_i + \Omega_j a_j)} \tag{13} \]

Special Case:

We know I_0(0)=1, so we get q=1 as a Special case.
Fig. 2, Fig. 3 shows the SOP vs Signal to Noise Ratio, where the unit of Cth is bits/s/Hz. Generally Nakagami-q span ranges from q=0 to 1. Where it exhibits one sided Gaussian noise at 0 and Rayleigh at 1. Fig 2. exhibits the values for q1=q2=q3=0.5. According the definition and meaning of secrecy outage probability with the strengthening of the signal i.e., increasing signal to noise ratio (SNR) values the outage probability will be reduced. We S-SU link is better than that of S-Eav link. Similarly fig 3 exhibits the values for q1=q2=q3=1. By adjusting the Cth values from 0.794 to 1 to 1.254 we achieved good results.

V. Conclusion

In this paper, the analytical model for the SOP of a basic underlay cognitive radio networks unit over Nakagami-q channel is presented, verified by simulation.

REFERENCES