Modeling and Optimal Control of a DC Motor

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Abstract: *Realization in the simulation environment in order to acquire the behaviors of the systems prior to the real-time studies is of great importance in terms of detecting the faults that may occur during the real-time studies and distinguishing them in the algorithm developing phases and preventing them. In this study; modeling, inspecting, and the effect of the kalman filter in the noise environment and comparing the performances of the filtered controllers of a direct current (DC) motor is shown. Linear quadratic regulator (linear quadratic regulator-LQR) and proportional integral differential (proportional-integral-derivative PID) methods among the optimal control techniques were used for the speed monitoring of the DC motor. Both process noise and computation noise were applied to the DC motor system. Kalman filter was designed to increase the performance of the controllers in the noise environment. The controller was designed in the MATLAB environment and according to the gathered simulation environment results, the applied control methods were compared and the results were scrutinized.*

Keywords — *Kalman Filter, LQR, PID, DC Motor*

I. INTRODUCTION

Direct current (DC) motor are commonly used today due to their features such cost-efficiency, ease of use, high performance, longevity and quiet operation. Dc motor are prevalently used in the areas such as space and computer technologies, defense industry, robotics applications and many more [1-6]. Various methods were used in the motor controls in recent years [7-8-9-10].

Kassem and Yousef [11], Sliding mode control method was used in the Dc motor speed monitoring. Dc motor was inspected by using the Gaddam and Akula [12], Fuzzy method, Wang [13] et al. Fuzzy-PID, Abut [14] genetic algorithm method and optimal control method, Weerasoorya and Al-Sharkawi [15] YSA method and El-Gammal and El-Samahy [16], particle cartload optimization methods. While the traditional Ziegler-Nichols method and modified Ziegler-Nichols method were compared in the PID controllers designed for the Dc motor system, it was found that the modified Ziegler-Nichols method was more successful [17]. The systems were examined noiseless or with just the computation noise generally in the studies carried out.

In this study, kalman based LQR and PID controllers were designed by using the state-space model of the system for the dc motor speed monitoring. Performances of the controllers were examined by adding both process noise and computation noise to the system. Performance of the controllers in the noise environment was improved with kalman filter design and the performances of the filtered inspectors were compared. Additionally, the experiment mechanism that will be used in the future studies is shown in the Figure 1.

Fig. 1 Photograph of the experimental setup

II. MATHEMATICAL MODEL OF DC MOTOR

Direct current motor are the most commonly used motors in the control systems. They may provide rotation and offset movement. DC motor model is shown in Figure 2.

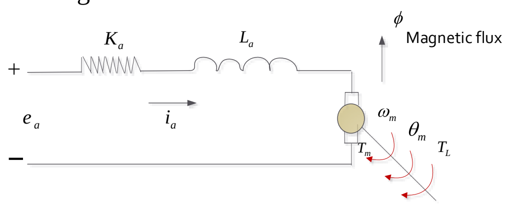

Fig. 2 Dc motor model

Moment received from the electric motor;

$$T_m(t) = K_m.\Phi.i_a(t) = K_i.i_a(t) \quad (1)$$

$$\frac{di_a}{dt} = \frac{1}{L_a}.e_a - \frac{R_a}{L_a}.i_a - \frac{1}{L_a}.e_b \quad (2)$$

$$T_m = K_i.I_a \quad (3)$$

$$e_b = K_b . \frac{d\theta_m}{dt} = K_b . \omega_m(t) \tag{4}$$

$$J_m \frac{d^2\theta_m}{dt^2} = T_m - T_L - B_m \frac{d\theta_m}{dt} \tag{5}$$

If we take our variables as i_a, θ_m and ω_m, the equations of state from the first order can be written as follows.

$$\dot{\underline{x}} = A . \underline{x} + b . u \tag{6}$$

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega_m}{dt} \\ \frac{d\theta_m}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ \frac{K_t}{J_m} & -\frac{B_m}{J_m} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} e_a - \begin{bmatrix} 0 \\ \frac{1}{J_m} \\ 0 \end{bmatrix} T_L(t) \tag{7}$$

$$y = Cx + Du \tag{8}$$

The parameters of the dc motor are shown in the Table 1.

TABLE I
PARAMETERS OF DC MOTOR

Sembol	Description	Units	Value
m	Body Mass	*kg*	10
J	Body Inertia	*kgm^2*	0.171
K_m	Motor Constant	*Nm/A*	3520
R_a	Motor Resistance	*Ohm*	55
l_o	Leg Length	*m*	0.323
L_m	Motor electric inductance	*H*	0.3
B_m	Damping ratio of the system friction constant	*Nm.*	0.097

III. THE CONTROLLER DESIGN

Different controllers are used in the control of the dc motor. The purpose of the control systems used is to make the value of the output of the system to follow the aimed (reference) value. Any difference between the aimed (reference) output value in the system and the existing system output value is considered as a fault. This fault is tried to be minimized via the controller applied to the system.

III. A. PID CONTROL

Even though the PID (proportional integral derivative) control method used in many practices is an old method, it displays a good performance [18]. As it is easier to adjust compared to the other controllers and it is a simple control mechanism, it is widely used. It was observed that it is not a vital necessity to know the mathematical model of the process that will be controlled by PID control completely and that the control parameters can be adjusted favorably. Basic structure of the PID control method can be seen in the Equation 9[19].

$$u(t) = K_p e\ t\ + K_I \int_0^\tau e\ t\ dt + K_D \frac{d}{dt} e\ t \tag{9}$$

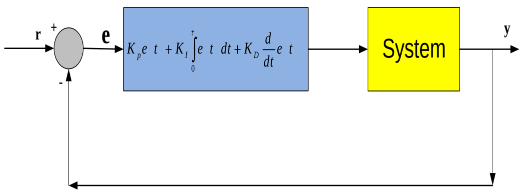

Fig. 3 Block diagram of PID feedback system

u, K_p, K_i, K_d, and e are named respectively as controller output, modulating gain, integral gain, differential gain and error signal. Block diagram of PID feedback system is shown in the Figure 3. In this study, Ziegler-Nichols method suggested by John G. Ziegler and Nathaniel B. Nichols is used to find the PID coefficients and closed loop control type is used within this method [20]. General control parameters of Ziegler-Nichols method is shown in the Table 2.

TABLE II
CONTROL PARAMETERS ACQUIRED VIA ZIEGLER-NICHOLS METHOD

Control	K_P	K_I	K_D
P	$0.5*K_{cr}$	∞	0
PI	$0.4*K_{cr}$	$0.8*P_{cr}$	0
PID	$0.6*K_{cr}$	$0.5*P_{cr}$	$0.125*P_{cr}$

III.B. LQR (Linear Quadratic Regulator)

LQR control method is the modern control method used to control a system [21]. This control method is widely used in the in optimal control problems in literature [22-24]. The purpose of control here is to minimize the quadric performance index.

$$u = -K * x \tag{10}$$

For the solution of the optimal control vector shown by the equation above, cost function should be minimized. Performance index with the help of the state-space equation.

$$J = \frac{1}{2} \int_0^\infty (x^T(t) Q x + u^T R u) dt \tag{11}$$

Performance index is a function and choosing the parameters of this function in a way that will make the function minimum or maximum makes the control system optimum. The value of the function indicates that the real performance of the system is how compatible to the desired performance. In other words, performance index is the measure of the deviation from the ideal performance. A fault that should be minimized with this index can be the integral of the fault function. And the ideal performance gets closer as the fault integral is minimized. The most basic control problem in the engineering is to determine the optimal control rule minimizing the performance index given under a variety of safety and economical constraints. In classic linear optimal control, the u(t) vector is chosen in a way that makes the performance index minimum. Usually the performance index selected in the system control is quadratic in terms of both x(t) and u(t). The integral of the total of the expressions the Q and R matrices are in is required to be the minimum. This means the minimizing of the equation 11. Here, the Q and R matrices are named as the weight matrices and Q is a matrix in at the size of [2Nx2N] and R is a positive matrix at the size of [mxm]. Q is a positive semi-defined symmetric matrix and R is a positive defined number ($Q \geq 0, R > 0$).

K optimum feedback gain matrix is calculated with the equation below:

$$K = T^{-1}\left(T^{T}\right)^{-1} B^{T} P = R^{-1} B^{T} P \quad (12)$$

The value of the P positive defined matrix is calculated with the help of Riccati equation.

$$A^{T}P + PA - PBR^{-1}B^{T}P + Q = 0 \quad (13)$$

Linear quadratic regulator (LQR) block diagram is shown in Figure 4.

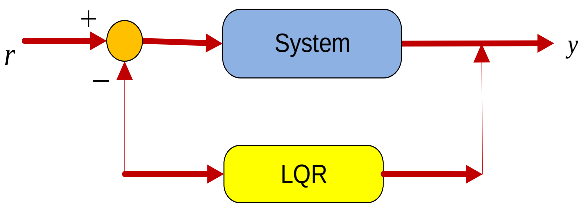

Fig. 4 Linear Quadratic Regulator Structure

IV. KALMAN FILTER DESIGN

Kalman filter is a filter estimating the status of the systems using the input and output information [25-26]. It has become one of the popular control methods for the DC motors without the transfer function or the state-space speed sensor. The inputs of the system are the armature voltage and armature current, system output is the speed of the dc motor in this system. Kalman filter is a kind of filter that can conduct optimally filter the operation and computation noise as long as the covariance of the noise are known. Estimation with this filter results in minimizing the covariance matrix of the fault for the systems exposed to Gauss computation or operation noise. Kalman estimator is shown in the equation number 5[27].

$$\dot{x} = Ax + Bu + w \quad (14)$$

$$y = Cx + v \quad (15)$$

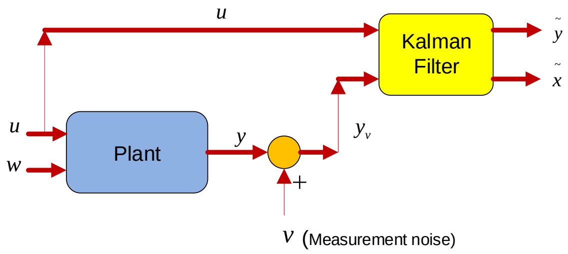

Fig. 5 Block diagram of the Kalman estimator

Respectively, w represents the operation noise given to the system randomly, and v represents the computation noise. In general, the following are the equations of Kalman estimator.

$$\dot{x} = Ax + Bu + w \quad (16)$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x}) \quad (17)$$

$\dot{\hat{x}}$ it is the estimated x value.

$$AP + PA^{T} - PC^{T}R^{-1}CP + Q \quad (18)$$

Q must be positive defined and R must be positive semi-defined and the system must be observable. Operation noise is expressed as w~N(0, Q) and measurement noise is expressed as v~N(0,Q). Here, Q is the input covariance matrix and R is the output covariance matrix. P algebraic invariant minimizing the cost function is calculated with Riccati equation. Filter gain, L, is calculated as shown with Riccati equation.

$$L = PC^{T}R^{-1} \quad (19)$$

$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and, $R = 1$, L value $L = 5.9 \quad 2.8$ is received. Figure 6 shows the block diagram of the kalman filter.

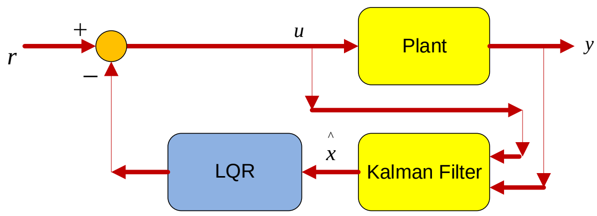

Fig. 6 Block diagram of the LQR control

V. NUMERICAL SIMULATION

In this section, the control of the dc motor is conducted using MATLAB/SIMULINK with kalman filter based controllers. Design, calculations and simulation of the LQR and PID controllers are done. Simulink model of the kalman filtered LQR controller is shown in Figure 6. Weight matrices used in the LQR control $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 1$,

L value $L = \begin{bmatrix} 5.9 & 2.8 \end{bmatrix}^{T}$ is received.

The control parameters used in controlling the dc motor with Kalman filtered PID controller is shown in Table 3.

TABLE IIII
PID CONTROL PARAMETERS

Control	K_P	K_i	K_D
PID	6.45	24.6	2.34

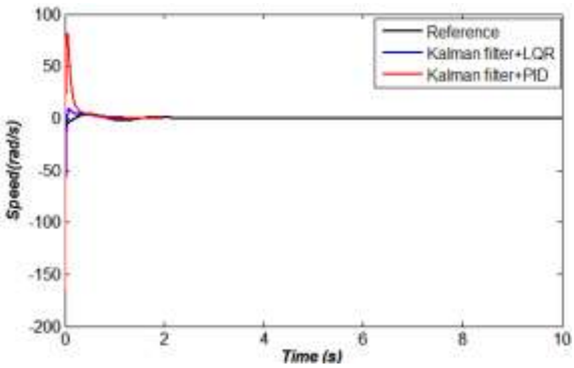

Figure 7: The speed results of numerical simulation

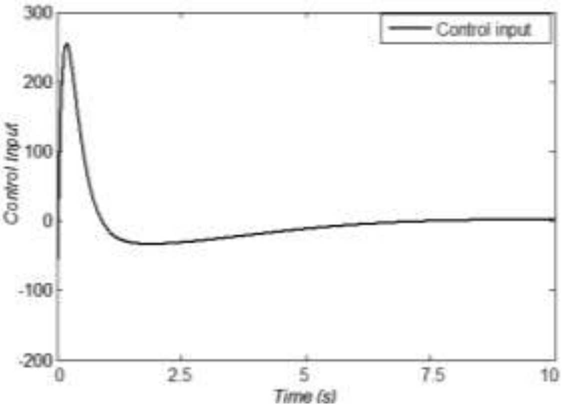

Figure 8: Control signal

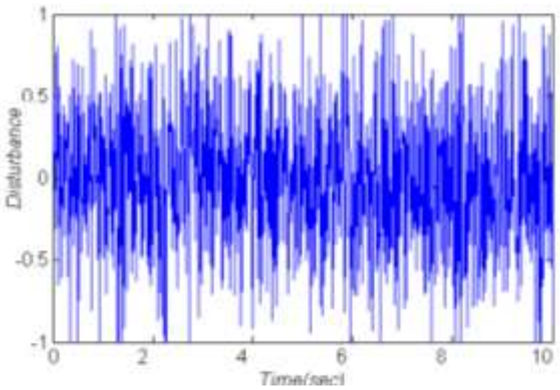

Figure 9: Disturbance input signal affecting the motor

Kalman-based PID and LQR control methods are compared in the Figure 7. According to the simulation results, the kalman filtered LQR control method followed the reference value better. Figure 8 shows the control signal. Figure 9 shows the disruptive input signal affecting the dc motor. Dc motor control was simulated using the parameters shown in the Table 1. Speed monitoring was conducted in the simulation studies and the performance was evaluated.

VI. CONCLUSIONS

In this study, LQR control design among the optimal control techniques and PID control design, one of the classic control methods, was conducted for the dc motor speed monitoring. Motor control was conducted in a noise environment. Kalman filter was designed to improve the performance of the controllers in the noise environment. The effect of kalman filter on the controllers is shown in the Figure 7. Kalman filtered LQR method and the kalman filtered PID controller were compared and it was observed that the kalman filtered LQR method is more successful. In the future studies, this control method will be applied to the experiment mechanism shown in the Figure 1 and the results will be compared.

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APPENDİX

Nomenclature

i_a	Motor current
R_a	Motor resistance
e_b	emf
T_L	Load torque
ϕ	Magnetic flux
J_m	Moment of inertia
B_m	Viscous damping coefficient
L_a	Motor inductance
e_a	Motor voltage
K_b	emf constant
θ_m	Angular rotation of rotor
K_i	Torque constant
w_m	Angular velocity of rotor