Comparative Analysis of Natural Frequencies of Beam Elements with Varying Cross Sections and end Conditions

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Abstract:

The present work highlights the variations of the Eigen values or otherwise the natural frequencies of cantilever beam and simply supported beam structures with respect to the beam cross sections and end conditions. NATURAL FREQUENCIES play an important role in the smooth operation of the element or structure. Natural frequencies have their importance in the field of dynamics and hence the study is called as Dynamic analysis.

The present study is divided into two categories viz,.

a) Behaviour of Natural frequencies with respect to varying depths and
b) Behaviour of Natural frequencies with respect to varying boundary conditions.

Graphs are drawn to indicate the behavior of the Eigen values for the above two cases. The depth of the beam considered for the analysis are 6 mm, 8 mm, 10 mm, 12 mm and 14 mm. Based on the type of the beam, the end condition depends. Here cantilever beams and simply supported beams are taken for analysis.

Key words: Eigen values, natural frequencies, dynamic analysis, beam vibrations, resonance etc

1. Introduction:

All the systems around us are subjected to some sort of vibrations or disturbances. Therefore they are said to be under vibrations. A body vibrates with some frequency which has to be determined. The fundamental frequency with which a body vibrates is called as the natural frequency. Other than the first natural frequency it vibrates at different frequencies which may be called as second natural frequency and so on. These natural frequencies are to be calculated and the system should be subjected to run at frequencies other than the natural frequencies. Only then the system will be safe and smooth for operation.

Resonance is a phenomenon when the frequency of excitation coincides with one of the natural frequencies of the system. At resonance, large oscillations occur resulting in the mechanical failure of the system resulting to heavy wear and tear and ultimately seizure of the equipment occurs. So the study of vibration is essential for an engineer to minimize the vibrational effects on the components and to take necessary precautions in designing them suitably [1].

The study of the vibrational behavior of the structure is called as Dynamic Analysis which involves evaluating the Eigen Values, Eigen Vectors, damping ratios, FRFs etc.[2]. In the present work two beam structures are taken for analysis namely cantilever beam and simply supported beam.

2. Objective of the analysis:

As already discussed in the introduction the natural frequency plays an important role for any structure or system to work smoothly and securely. The aim of this work is to show how the Eigen Values vary for different types of cross sections and end boundary conditions.
2.1 The first part of the work is on the specimens with varying cross sections. To be very alert and safe, the cross sections are not vaguely chosen for analysis. A study is done to decide the affecting parameter or dimension for the Natural frequencies.

For a specimen with a rectangular cross section, the three varying parameters are length, width or breadth and depth or thickness or height.

Needless to mention the necessity of using only a rectangular cross section than square is that there will be no scope for variation of width and depth in case of square cross section, as both will be of same dimension. The other influencing factor is that generally rectangular cross sections are used for the feasibility of space and load take-up.

2.2 Among the parameters of the beam, the length of the beam structure will be a constant in practical applications. As far as the width is concerned, it has scarcely any effect on the natural frequency with its variation. This is decided by assigning arbitrary values for 3 parameters viz, length, width and depth, keeping 2 of the parameters as constants and varying the other to find the natural frequencies.

The Fig.1(a),(b) and (c) are drawn by considering the dimensions of the cantilever beam just to ascertain the effect on the natural frequency when two parameters are kept constant and one of them is varied.

The graphs indicate that the thickness or depth and the length of the specimen are the affecting parameters for the natural frequencies. Hence criteria to decide the parameters depend on the above analytical results. So length and depth of the specimen are to be varied as they affect the natural frequency to a greater extent. On the contrary, the variation in the width of the beam will negligibly affect the natural frequency as seen from the graph 2(c).

Generally for any structure the impact load is the criterion which primarily depends on the depth than on the length of the beam. Hence length is kept as a constant in the present problem. The only left out factor is the depth or thickness which is ultimately considered for variation.

2.3 The second part of the work is on two different end conditions viz., Cantilever and simply supported ends. The behavior of the Natural frequencies of the above beam structures is analysed.

3. MATLAB Application:

MATLAB software has been used for the calculation and this has eased the work to a maximum extent by avoiding laborious and tedious calculations. The extensive use of this software has helped to achieve results to a...
greater accuracy with less time consumption. This software is more feasible for all the operations of matrices. Exclusive feature of this software is that there is no restriction on the order of the matrices for any operation to be carried out. As there are direct functions available in the software the syntax in the help menu can be used to run the program.

As far as this work is concerned the function for Eigen value and Eigen vector are also directly available from which the natural frequencies are calculated. Previous researchers also have used this Software for their calculation [3].

4. Specimen Details:

The details of the test specimen shown in Fig. 2 are as follows

4.1 Material: Mild Steel

4.2 Properties: Young’s Modulas of elasticity = 2.1*10^11 N/mm²

Material Density = 7850 kg/m³

Length = 0.5 m

Width = 0.048 m

Depth = Varying parameter (range 6 mm, 8 mm, 10 mm, 12 mm and 14 mm)

![Fig. 2](image)

4.3 End Conditions:

(i) Cantilever beam: one end fixed and other end is free.

(ii) Simply supported beam: both ends kept on supports.

4.4 Boundary Conditions:

For Case (i) V₁ = 0, (ii) θ₁ = 0, (iii) V₂ = Unrestrained and (iv) θ₂ = Unrestrained

Case (ii) V₁ = 0, (ii) V₂ = 0, (iii) θ₁= Unrestrained and (iv) θ₂ = Unrestrained

5. Validation of the result:

The natural frequencies of cantilever beam are evaluated for the above dimensions by using the following formulae:

5.1 Analytical equations and calculations:

The first natural fundamental is given by,

ωₙ₁=1.875² × √(EI/ρA²) [4]

Where, l=bd³/12

And A=bxld

Similarly the second natural frequency ωₙ₂=4.694² × √(EI/ρA²) [4]

5.2 FEM results using MATLAB:

A program is designed to calculate the natural frequencies of the cantilever beam in MATLAB and the results are obtained. The basic characteristic equation according to FEM to calculate the natural frequency is |K-λM|=0, where K is the stiffness matrix, which is symmetric matrix reflecting the property of the material, λ is the Eigen value=ω² and ω = 2πf and M is the mass matrix [5].
5.3 Experimental Validation:

The test specimen is taken for vibration analysis and fixing one end rigidly, the test piece acts a Cantilever beam. Test sensors are fixed and the frequency is directly read on the analyzer. The experimental set up is shown Fig 3 and Fig 4

Fig. 3: Clamping of the test specimen as cantilever beam Fig. 4: Specimen fixed with sensors

(The above experimental work is carried out at Dynamics Laboratory, Data Physics (Bharat) Pvt. Ltd., Bangalore)

The value of \( f_1 \) and \( f_2 \) by experimental analysis is 18.50 hz and 116.50 hz. This is read from the Fig. 5

Fig. 5: The natural frequencies of the cantilever specimen obtained experimentally

The values of the first and second natural frequencies obtained by the three methods are tabulated in the Table 1

<table>
<thead>
<tr>
<th></th>
<th>Analytical value</th>
<th>FEM value</th>
<th>Experimental value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>20.050 hz</td>
<td>20.131 hz</td>
<td>18.50 hz</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>125.66 hz</td>
<td>198.593 hz</td>
<td>116.50hz</td>
</tr>
</tbody>
</table>

The first natural frequency also called as FUNDAMENTAL FREQUENCY is of primary importance.

It is seen that the values obtained by analytical and FEM calculations are a little higher than experimental values. This is due to the under estimation of the global mass matrix and over estimation of global stiffness matrix as substantiated in reference [6].Thus the analytical results are validated experimentally.
6. Results and Inferences:
6.1 Variations of natural frequencies of a cantilever beam with respect to depth:

The Eigen Values and the corresponding Natural Frequencies of a Cantilever Beam with varying depths are tabulated below in table 2. For a reasonable comparison only first two natural frequencies are taken for consideration.

<table>
<thead>
<tr>
<th>Depth(mm)</th>
<th>( \lambda_1 )</th>
<th>( f_1 ) (hz)</th>
<th>( \lambda_2 )</th>
<th>( f_2 ) (hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.0160*10^6</td>
<td>20.131</td>
<td>1.5557*10^6</td>
<td>198.51</td>
</tr>
<tr>
<td>8</td>
<td>0.0285*10^6</td>
<td>26.868</td>
<td>2.7657*10^6</td>
<td>264.680</td>
</tr>
<tr>
<td>10</td>
<td>0.0445*10^6</td>
<td>33.573</td>
<td>4.3213*10^6</td>
<td>330.847</td>
</tr>
<tr>
<td>12</td>
<td>0.0641*10^6</td>
<td>40.295</td>
<td>6.2227*10^6</td>
<td>397.014</td>
</tr>
<tr>
<td>14</td>
<td>0.0873*10^6</td>
<td>47.024</td>
<td>8.4698*10^6</td>
<td>463.187</td>
</tr>
</tbody>
</table>

The graph of the variation of the first and second natural frequencies with respect to depth of the specimen is shown in Fig. 6 and Fig. 7 respectively.

Fig. 6: Variation of first natural frequency versus depth
Inference: It is seen from the above graph that the curve is a straight line for both I and II natural frequencies. So the natural frequencies for a cantilever beam vary directly with respect to the variation in the depth of the beam.

6.2 Variations of natural frequencies of a Simply Supported beam with respect to depth:

The Eigen Values and the corresponding Natural Frequencies of a Simply Supported Beam with varying depths are tabulated in table 3. For a reasonable comparison only first two natural frequencies are taken for consideration:

<table>
<thead>
<tr>
<th>Depth(mm)</th>
<th>λ₁ (Hz)</th>
<th>f₁ (hz)</th>
<th>λ₂ (Hz)</th>
<th>f₂ (hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.1541*10^6</td>
<td>62.475</td>
<td>3.2359*10^6</td>
<td>286.297</td>
</tr>
<tr>
<td>8</td>
<td>0.2739*10^6</td>
<td>83.300</td>
<td>5.7527*10^6</td>
<td>381.730</td>
</tr>
<tr>
<td>10</td>
<td>0.4280*10^6</td>
<td>104.121</td>
<td>8.9885*10^6</td>
<td>477.159</td>
</tr>
<tr>
<td>12</td>
<td>0.6160*10^6</td>
<td>124.950</td>
<td>12.9430*10^6</td>
<td>572.581</td>
</tr>
<tr>
<td>14</td>
<td>0.8390*10^6</td>
<td>145.775</td>
<td>17.6180*10^6</td>
<td>668.033</td>
</tr>
</tbody>
</table>

The above results of the simply supported beam are cross checked analytically with the help of the formulae given in the reference [7]

The graph of the variation of the natural frequencies with respect to depth of the specimen is shown below.
Fig. 8: Variation of first natural frequency wrt depth

Fig. 9: Variation of second natural frequency versus depth

Inference: It is seen from the above graph that the curve is a straight line for both I and II natural frequencies. So the natural frequencies for a simply supported beam vary directly with respect to the variation in the depth of the beam.
7. **Conclusions:**

From the above analysis it is clear that the values of the natural frequencies vary directly as the depth of the specimen for both the cantilever and simply supported end conditions i.e. the curve is a perfect straight line.

It is seen that the value of both first and second natural frequencies of a simply supported beam are much higher than cantilever beam for the same dimensional parameters.

**References:**


3. Dr. Luay S. Al-Ansari 2012 *Calculating of Natural Frequency of Stepping Cantilever Beam* IJMME-IJENS Vol 12 pp 59-68.


