Outage Performance of Dual-Hop Parallel AF Relayed FSO Systems

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Abstract—In this paper, we analyze the expressions for the performance of dual-hop parallel relayed free space optical communication systems in the presence of atmospheric turbulence, misalignment fading or pointing errors and path loss. The parallel relay links follow the amplify and forward protocol and assume that the turbulence channels modelled with gamma-gamma fading statistics are independent but might or might not be identically distributed(i.i.d). Further, Max-Select relay selection scheme has been considered for parallel optical links under the assumption that all the relays are active and full channel state information is available at transmitting as well as receiving terminals. Based on the analytical expressions for the moment generating function and cumulative distribution function for single dual-hop link, the expression for outage probability of many parallel relayed dual-hop links is derived for variable gain and fixed gain relays based on max-select relay selection scheme.

keywords: Amplify and forward relay, Fixed gain relays, Gamma-Gamma distribution, Meijer’s G-function, Moment generating function, Variable gain relays.

I. INTRODUCTION

The Free-space optical communication (FSO) refers to the transmission of data optically through the atmosphere with the help of optical transmitting devices such as lasers or light emitting diodes. The FSO is becoming highly popular in the field of research due to its many advantages over the conventional forms of communication [1]. It caters to the demand of high bandwidth, is robust to electromagnetic interferences, provides a reliable solution for first-mile and last-mile infrastructure problems, has an unregulated spectrum and can be virtually reused. It offers low cost of installation and a much secured form of communication as compared to radio-frequency (RF) links. Thus, the FSO communication can be used as an alternative technology or as a complementary technology to its RF counterparts.

However, with many features comes along few problems in the FSO systems which could lead to errors in the data sent or complete loss of data in few situations. These links are vulnerable to adverse channel conditions that could be due to rain, fog, snow, dust, atmospheric turbulence and pointing errors as well [2]. The atmospheric turbulence is caused by the variations in the refractive index of the air molecules due to varying temperature and moisture after every few metres, whereas, the misalignment of the source and the destination transceivers due to the sway of high rise buildings causes pointing errors. These aforementioned factors could lead to fading and an increase in Bit Error Rate (BER) [3] - [5].

Thus, to combat with the effects of atmospheric turbulence and other losses, research has been carried out in the fields of technologies like multiple input multiple output (MIMO) and cooperative diversity (CD). In the CD, many different direct or indirect links can be considered from source to destination with the help of relays to increase the distance, reduce errors and utilize the advantage of cooperation to the most [6].

The relaying FSO systems could work either on one or both of the two practical schemes which are decode-and-forward (DF) and amplify-and-forward (AF). In the AF relaying scheme also know as non-regenerative systems, the relay simply forwards the received signal to the next relay or the destination without performing any decoding functions. It just amplifies the signal and forwards whereas in the DF relaying scheme also known as regenerative systems, the relay first decodes the received signal, then modulates it and forwards it to the next relay or the destination if the SNR ratio is high. Since AF relaying systems do not perform any decoding operation or modulation, they consume less power as compared to DF relaying systems [7]. Although FSO systems can work much more efficiently in cooperative environment with suitable relaying schemes, but with the past research it has been found that there could be issues in synchronization between the relay nodes and inefficient utilization of resources affecting the relaying paths. In that case, selective protocols can be used that selects the best the relaying node with minimum error suitable for forwarding the received signal further [8]. The selective relay protocols are further classified into best relay selection and partial relay selection. In the former selection scheme, the node is elected on the basis of channel state information (CSI) between source to relay and relay to destination whereas only the source to relay is considered in the latter selection scheme. CSI assisted relays collect the channel information from the previous hop to control the relay gain so that the relay can perform better [9].

AF decoding scheme has been discussed for both serial and parallel relaying. It uses intensity modulation with direct detection (IM/DD) [10]- [12]. With the study of previous
work, it was found that the systems could not give exact analysis of performance as the end-to-end signal to noise ratio (SNR) of the system was upper bounded using the harmonic geometric mean (HGM) inequality. But we could find out the exact performance of the FSO systems with the help of special functions such as bivariate Meijer G-function (BMGF) where much work has not been done in past. Based on the above literature survey, we model a dual hop parallel relaying system based on AF relaying protocol consisting of many relays connected between source to destination without any direct link between the source to destination. The atmospheric turbulences and misalignment fading will be modelled using Gamma-Gamma distribution. Further we will calculate the exact performance expressions for cumulative distribution function (cdf) for parallel relays connecting source to destination. Further, based on the results of the aforementioned expressions, we will derive the outage probability. Finally, appropriate solution will be obtained in terms of MATLAB software package.

II. SYSTEM MODEL

The system model consists of a source and destination through which the data has to be shared. We assume that the distance between the source and the destination is very large due to which line-of-sight communication is not possible between them and so we will make use of relay nodes connected in parallel that will help the source in transmitting the data to the destination. The source and the destination consists transceivers of single directional apertures whereas the relays consist of transceivers with two directional apertures out of which one points towards the source and the other towards the destination respectively. The source selects the best relay node to transmit its data out of all the nodes in parallel relaying. The relay node selected by the help of relay node selection process further transmits data to destination by using amplify and forward protocol. The system is based upon SIM (Subcarrier Intensity Modulation) and direct detection method. The AF relays are CSI assisted which assumes that the signal transmitted by the source has average power normalised to unity. Also, it is assumed that the CSI is available at the receiving terminals, that is, at the destination and at the relay nodes.

The channel coefficient for source to relay link and relay to destination is based on the combined effects of various channel impairments such as path loss \((h_t)\), atmospheric turbulence induced scintillation \((h_a)\) and misalignment fading \((h_p)\) due to pointing errors and is given as

\[ h_{xy} = (h_t) \times (h_a) \times (h_p). \]

The Path Loss \((h_t)\) is represented using the Beers-Lambert Law in [13] which can be written as \(h_t = \exp(-\sigma L)\) where \(\sigma\) denotes the attenuation coefficient. Another type of channel impairment is Atmospheric Turbulence \((h_a)\) and here we consider that the turbulence is induced due to Scintillation which is resulted due to the formation of eddies that are formed by different atmospheric conditions like moisture and refractive indices of air molecules at different spots in environment and cause fading. The pdf for turbulence induced scintillation is given as

\[ f_{h_a}(h_a) = \frac{2(a \beta H_a^2)}{(\alpha \beta H_a^2)(\alpha H_a^2)^{a+1}} K_{a-\beta}(2\sqrt{\alpha h_a}) \]

Here, \(K_a\) expresses the modified Bessel function of second kind and order \(a\), whereas, \(\alpha\) and \(\beta\) are turbulence parameters both being greater than zero and representing small-scale and large-scale eddies in the atmosphere respectively. The final impairment that is considered is Pointing Error Losses \((h_p)\) that is caused due to misalignment of the transceivers at the source to relay end or the relay to destination end. This kind of loss is modeled using Rayleigh distribution given in [13] as

\[ f_{h_p}(h_p) = \frac{\xi^2}{A_0 h_p^{\alpha-1}}, 0 \leq h_p \leq A_0 \]

where, \(A_0\) is the equivalent beam width at the receiver and \(\xi\) is the standard deviation of the pointing error displacement at the receiver. Thus, the combined channel pdf as given in [14] where \(xy\) represents the relay link nodes with \(\{x\} \in \{s,r\}\) and \(\{y\} \in \{r,d\}\) is

\[ f_{h_{xy}}(h) = \frac{\alpha \beta \xi^2}{A_0 h_t^{\alpha} \Gamma(\alpha) \Gamma(\beta)} \times G_{1.3}^{3.0} \left( \frac{\alpha \beta}{A_0 h_t^{\alpha}}, h, \frac{\xi^2}{2 \alpha - 1, \beta - 1} \right) \]

Now, the instantaneous SNR for source to relay and relay to destination link is given as

\[ \gamma_{xy} = \frac{\eta |h_{xy}|^2}{\sigma_{n,xy}^2} \]

Here, \(\eta\) is the optical to electrical conversion ratio and \(\sigma_{n,xy}\) for \(\{y\} \in \{r,d\}\) is the variance of additive white Gaussian noise (AWGN) at the relay and at the destination respectively. By the power transformation of 1 with the help of equation given in [15], we get the pdf of instantaneous SNR as

\[ f_{\gamma_{xy}}(\gamma_{xy}) = \frac{\xi_{xy}^2}{2 \gamma_{xy}^{\alpha_{xy}} \beta_{xy}^{\beta_{xy}}} \times G_{1.3}^{3.0} \left( \alpha_{xy} \beta_{xy} \sqrt{\frac{\xi_{xy}}{\Omega_{xy}}}, \frac{\xi_{xy}^2 + 1}{\Omega_{xy} \xi_{xy}^{-2}, \alpha_{xy}, \beta_{xy}} \right) \]

where \(\Omega_{xy}\) denotes the average electrical SNR and \(G_{p,q}^{m,n} (\cdot, \cdot)\) is the Meijer’s G-function given in [16], eq(8.2.1).

III. MAX-SELECT PROTOCOL

The Max-Select protocol is a relay selection scheme in which a single relay is selected out of multiple relays that are available. The method for is to check the end-to-end SNR of all the relay links and to select the i-th link with maximum end-to-end SNR. The selection of the relay is done based on the rule

\[ \gamma_{eqi}^{max} = \arg \max \{\gamma_{eqi}\} \]

where \(i \in \{1, 2, \ldots N\}\) as independent random variables. The selection process is carried out by evaluating the Channel State Information (CSI) of all source to relay and relay to destination links.
IV. OUTAGE ANALYSIS

A. Variable Gain Relays

The equivalent end to end SNR $\gamma_{eq}$ for s-r and r-d links with kth number of variable gain parallel relays from source to destination is given as

$$\gamma_{eq}^v = \frac{\gamma_{srk}\gamma_{rkd}}{\gamma_{srk} + \gamma_{rkd}} = \left(\frac{1}{\gamma_{srk}} + \frac{1}{\gamma_{rkd}}\right)^{-1} \tag{5}$$

where $\gamma_{srk}$ is the instantaneous SNR of s-r link with kth number of relays and $\gamma_{rkd}$ is the instantaneous SNR of r-d link with kth number of relays.

Now, the MGF of $\gamma_{eq}^v$ for variable gain relay links is given in [17, Eq(3)] can be modified and written as

$$\mathcal{M}_{\gamma_{eq}^v}(s) = 1 - 2\sqrt{s} \times \int_0^\infty J_1(2\sqrt{s}p)\mathcal{M}_{\gamma_{srk}^{-1}}(p^2)\mathcal{M}_{\gamma_{rkd}^{-1}}(p^2)dp$$

The formula for MGF of reciprocal of $\gamma_{xy}$ is given in [17] as

$$\mathcal{M}_{\gamma_{xy}^{-1}}(s) = \int_0^\infty \exp\left(-\frac{s}{\gamma_{xy}}\right)f_{\gamma_{xy}}(\gamma_{xy})d\gamma_{xy}$$

By substituting the value of $f_{\gamma_{xy}}(\gamma_{xy})$ in the above equation for $\mathcal{M}_{\gamma_{eq}^v}(s)$ and by using the identity given in [18, Eq(11)] and integrating it by using [18, Eq(21)] we get

$$\mathcal{M}_{\gamma_{eq}^v}(s) = \frac{2^{1+\alpha_{srk}+\beta_{srk}+\beta_{rkd}}}{4\pi\Gamma(\alpha_{srk})\Gamma(\beta_{rkd})} C_{2,7}^{7,0}\left(\frac{\alpha_{srk}^2\beta_{rkd}^2}{16}\Omega_{srk}\varphi_1;\varphi_2;\varphi_3\right) \times \left[\frac{\xi_{srk}^2}{2},\frac{\xi_{srk}^2+1}{2},\frac{\alpha_{srk}^2+1}{2},\frac{\beta_{rkd}^2+1}{2},0\right]$$

Finally, we put the derived value of $\mathcal{M}_{\gamma_{eq}^v}(s)$ in (6) and write it in terms of Meijer’s G-function as

$$\mathcal{M}_{\gamma_{eq}^v}(s) = 1 - 2\sqrt{s} \times \frac{\xi_{srk}^2\xi_{rkd}^2}{16\pi^2\Gamma(\alpha_{srk})\Gamma(\beta_{srk})\Gamma(\beta_{rkd})} C_{2,7}^{7,0}\left(\frac{\alpha_{srk}^2\beta_{srk}^2}{16}\Omega_{srk}\varphi_1;\varphi_2;\varphi_3\right) \times \int_0^\infty J_1(2\sqrt{s}p)C_{2,7}^{7,0}\left(\frac{\varphi_1p^2}{\Omega_{srk}};\varphi_2;\varphi_3\right)\times C_{2,7}^{7,0}\left(\frac{\varphi_1p^2}{\Omega_{srk}};\varphi_5;\varphi_6\right)dp \tag{8}$$

where

$$\varphi_1 = \left\{\frac{\xi_{srk}^2}{16},\frac{\xi_{srk}^2+1}{2},\frac{\alpha_{srk}^2+1}{2},\frac{\beta_{srk}^2+1}{2},0\right\},$$

$$\varphi_2 = \left\{\frac{\xi_{srk}^2}{2},\frac{\xi_{srk}^2+1}{2},\frac{\alpha_{srk}^2+1}{2},\frac{\beta_{srk}^2+1}{2},0\right\},$$

$$\varphi_3 = \left\{\frac{\xi_{srk}^2}{2},\frac{\xi_{srk}^2+1}{2},\frac{\alpha_{srk}^2+1}{2},\frac{\beta_{srk}^2+1}{2},0\right\},$$

$$\varphi_4 = \left\{\frac{\xi_{srk}^2}{2},\frac{\xi_{srk}^2+1}{2},\frac{\alpha_{srk}^2+1}{2},\frac{\beta_{srk}^2+1}{2},0\right\},$$

$$\varphi_5 = \left\{\frac{\xi_{srk}^2}{2},\frac{\xi_{srk}^2+1}{2},\frac{\alpha_{srk}^2+1}{2},\frac{\beta_{srk}^2+1}{2},0\right\},$$

$$\varphi_6 = \left\{\frac{\xi_{srk}^2}{2},\frac{\xi_{srk}^2+1}{2},\frac{\alpha_{srk}^2+1}{2},\frac{\beta_{srk}^2+1}{2},0\right\}.$$

If we multiply two Meijer’s G functions, we can write the above equation in terms of BMGF by using the identity defined in [19, Eq(11)] as

$$\mathcal{M}_{\gamma_{eq}^v}(s) = 1 - 2\sqrt{s} \times \frac{\xi_{srk}^2\xi_{rkd}^2}{16\pi^2\Gamma(\alpha_{srk})\Gamma(\beta_{srk})\Gamma(\alpha_{rkd})\Gamma(\beta_{rkd})} \times \int_0^\infty J_1(2\sqrt{s}p)C_{2,7}^{7,0}\left(\frac{\varphi_1p^2}{\Omega_{srk}};\varphi_2;\varphi_3\right)\times C_{2,7}^{7,0}\left(\frac{\varphi_1p^2}{\Omega_{srk}};\varphi_5;\varphi_6\right)dp \tag{9}$$

where $S[\cdot;\cdot;\cdot]$ denotes the BMGF.

By integrating (9) with the help of identity given in [20, Eq(3.4)], we derive the MGF of equivalent end-to-end SNR, $\gamma_{eq}$, for kth number of parallel links from source to relay and relay to destination $\mathcal{M}_{\gamma_{eq}^v}(s)$ in closed form as

$$\mathcal{M}_{\gamma_{eq}^v}(s) = 1 - \frac{\xi_{srk}^2\xi_{rkd}^2}{16\pi^2\Gamma(\alpha_{srk})\Gamma(\beta_{srk})\Gamma(\alpha_{rkd})\Gamma(\beta_{rkd})} \times S$$

The cdf for equivalent end-to-end SNR, $F_{\gamma_{eq}}(\gamma)$, is defined as

$$F_{\gamma_{eq}}(\gamma) = \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{M}_{\gamma_{eq}^v}(s)\right]$$

where $\mathcal{L}^{-1}(\cdot)$ denotes the inverse Laplace Transform operator. So, by substituting the value of (10) in the above equation and with the help of the identity given in [20, Eq(2.7)] we get the exact and closed from expression of cdf of equivalent end-to-end SNR between source to relay and relay to destination for variable gain links as

$$F_{\gamma_{eq}^v}(\gamma) = 1 - \frac{\xi_{srk}^2\xi_{rkd}^2}{16\pi^2\Gamma(\alpha_{srk})\Gamma(\beta_{srk})\Gamma(\alpha_{rkd})\Gamma(\beta_{rkd})} \times S$$

The cdf of equivalent end-to-end SNR $\gamma_{eq}$ where $i \in \{1, 2, \ldots, N\}$ as independent random variables

$$F_{\gamma_{eq}^v}(\gamma) = F_{\gamma_{eq}^v}(F_{\gamma_{eq}^v}(\gamma_1) \land \cdots \land F_{\gamma_{eq}^v}(\gamma_N)) \tag{12}$$
So, the cdf of highest equivalent SNR for independently distributed variable gain links is given as
\[
F_{\gamma_{eq}^{max}} = F_{\gamma_{eq1}}(\gamma)F_{\gamma_{eq2}}(\gamma)\ldots F_{\gamma_{eqN}}(\gamma)
\]
\[
= \prod_{i=1}^{k} F_{\gamma_{eqi}}(\gamma)
\]
(13)

Similarly, the cdf of highest equivalent SNR for independently and identically distributed variable relay links is given by
\[
F_{\gamma_{eqi}^{max}} = [F_{\gamma_{eqi}}(\gamma)]^k
\]
(14)

\[B. \ Fixed \ Gain \ Relays\]

The equivalent end to end SNR \(\gamma_{eq}\) for s-r and r-d links with kth number of fixed gain parallel relays from source to destination is given as
\[
\gamma_{eq} = \frac{\gamma_{srk} \gamma_{rkd} \gamma_{eqfg}}{\gamma_{srk} + \gamma_{rkd} + C}
\]
(15)

where \(\gamma_{srk}\) is the instantaneous SNR of s-r link with kth number of relays, \(\gamma_{rkd}\) is the instantaneous SNR of r-d link with kth number of relays and C denotes the gain constant.

The MGF of \(\gamma_{eq}\) for fixed gain kth relays is given as
\[
M_{\gamma_{eqf}}(s) = 1 - 2\sqrt{s}
\times \int_{0}^{\infty} J_{1}(2\sqrt{sp})M_{\gamma_{srk}}(p^2)M_{\gamma_{rkd}}(p^2)dp
\]

The MGF for \(C_{\gamma_{srk} \gamma_{rkd}}\) is given in as
\[
M_{C_{\gamma_{srk} \gamma_{rkd}}}(s) = \int_{0}^{\infty} \int_{0}^{\infty} \exp\left(-sC_{\gamma_{srk} \gamma_{rkd}}\right) f_{\gamma_{srk}}(\gamma_{srk}) f_{\gamma_{rkd}}(\gamma_{rkd}) d\gamma_{srk} d\gamma_{rkd}
\]

So, by using the identity in [21], we obtain the MGF for \(C_{\gamma_{srk} \gamma_{rkd}}\) as
\[
M_{C_{\gamma_{srk} \gamma_{rkd}}}(s) = \frac{2^{\alpha_{srk} + \beta_{srk} + \alpha_{rkd} + \beta_{rkd} - 1}}{16 \pi^2 \Gamma(\alpha_{srk}) \Gamma(\beta_{srk}) \Gamma(\alpha_{rkd}) \Gamma(\beta_{rkd})} \phi
\]
\[
\phi = \frac{\xi_{srk}^2 \xi_{rkd}^2}{\xi_{srk}^2 \xi_{rkd}^2 + \xi_{srk}^2 \xi_{rkd}^2 + \frac{1}{2}} \Phi_1, \Phi_2, 0,
\]
(17)
The cdfs of highest equivalent SNR for independently distributed links and for independently and identically distributed links fixed gain relays are similar to variable gain relays as given in (13) and (14).

Cdf for independently distributed fixed relay links is

\[ F_{\gamma_{eqi}^f,g} = F_{\gamma_{eq1}}(\gamma) F_{\gamma_{eq2}}(\gamma) \ldots F_{\gamma_{eqN}}(\gamma) \]

where \( i \in \{1, 2, \ldots N\} \) as independent random variables. Cdf for independently and identically distributed fixed relay links is

\[ F_{\gamma_{eqi}^f,g} = [F_{\gamma_{eq}}(\gamma)]^k \]

(23)

V. NUMERICAL RESULTS

In this section, we illustrate performance of the FSO system when number of relays are used employing AF relaying, the effects of turbulence on the links between source to destination and pointing errors through numerical plots. In Fig.1, the Outage Probability versus Average SNR for multiple parallel relay links using AF relaying protocol is investigated. As we increase the number of parallel relays between source to destination, the outage performance increases. The threshold value is kept constant at 0.5 dB. It has also been observed that lower the threshold SNR, better is the outage performance. Also, the relay is considered to be equidistant from both source and destination.

In Fig.2, the Outage Probability of the system is plotted for two different values of \( K \) along the link distance, where \( K \) is the number of parallel relays. The total link length of \( L = 1800m \) is taken. It is observed that the best error performance is obtained when the distance of relay from both source and destination is equal. Also, as we increase the number of relays from 3 to 5, there is an increase in outage performance as now the source has more options to select from the multiple relays to transmit data using the maximum selection protocol. The threshold value and average SNR values are kept constant at 0.5dB and 30dB whereas the number of relays are increased.

In Fig.3, we study the outage performance of the FSO system with different turbulence conditions and different values for pointing errors. The threshold SNR is kept fixed at 1.5dB and the number of relays used are taken to be 4. The value of \( C_n^2 \) is \( 3 \times 10^{-14} \) and \( 1 \times 10^{-13} \) for moderate and strong turbulence conditions respectively whereas the values of \( \xi \) or pointing errors are taken to be 1.5 or 4. It is observed in the figure that with the increase in atmospheric turbulence and/or pointing errors, the outage performance of the system variably decreases.

VI. CONCLUSION

In this paper, we derived the exact closed form expression for CDF of dual-hop variable gain as well as fixed gain \( K \) number of parallel relay links based on Amplify and Forward relaying protocol. Based on the expressions for CDF, the outage performance of the relay links was analysed with respect to average SNR and link distance which is the distance between the source to relay and relay to destination. Also, Max-Select relay selection scheme was utilised to
select the best relay out of all parallel relays with maximum equivalent SNR so that adverse effects of pointing errors, path loss and turbulence are minimum on the FSO system performance.

REFERENCES


