Broad-Spectrum Scrutiny on Elegant Graphs

M.Sophia¹, M.Sangeetha²

¹Associate Professor, Department of Mathematics, Jeppiar Institute of Technology,
²Assistant Professor, Department of Computer Science And Engineering, Panimalar Engineering College, Chennai, India.

Abstract A graph G of m edges is referred as elegant if its vertices are labeled with distinct integer numbers like (0, 1, 2…m) such that the set of values on the edges acquired by the sums (mod m+1) of the labels of their end vertices is (1, 2, ..., m). Labeling must be distinct and should be non zero. In this paper, the broad spectrum of elegant graphs has been discussed and scrutinized.

Keywords Elegant labeling, regular graph, pseudo odd edges labeling, functions, complete binary tree.

I. INTRODUCTION

A sample graph with E edges is called elegant if the vertices of G can be labeled with distinct integer (0, 1, 2…R) in such a way that the set of values on the edges obtained by the sums mod (E+1) of the labels of their end vertices is {1, 2, ..., E}

In this paper, we refer a graph as an undirected graph without any loops or multiple edges and the cycle on n vertices are denoted by Cn and the path on n vertices by Pn Also special classes of graphs are discussed and more general consequences are scrutinized for elegant graphs. Along with these results of elegant graphs, adding the edge labeling and pseudo odd edge labeling are discussed with their related theorems.

II. GROUNDWORK

Definition: 1
An edge coloring of a graph is said to be optimal (or classical coloring) if no two edges incident with the same vertex, have the same color, and the minimum number of colors is used.

Definition: 2
An edge coloring of a graph is said to be Omni color if every edge has a dissimilar color.

Definition: 3
The family Mn of graph G is such that there exists an optimal edge coloring of the absolute graph Kn in which G appear as a omni colored sub graph (i.e.) every edge of G has dissimilar color.

Definition: 4
A connected graph with m edges is called harmonious if it is possible to label its vertices with distinct numbers (mod m) in such a way that the values on the edges obtained by sums (mod m) of their end points lab lings are also distinct (2).

III. ELEGANT LABELINGS AND EDGE COLORING

Initially, we prove the existence of a coloring at Kn with a omni colored path on n vertices as sub graph, we had been conjectured by Hartman [2]. In the second part we prove that the cycle on n vertices is elegant if and only if n≠1 mod(4) and give a new assembly of an elegant labeling of the path Pn, n≠4

Theorem: 2.1.1
The path on n vertices belongs to Mn if and only if n≠4, 6

Proof: Case 1
In the classical coloring of the edges of Kp+1 with 2p+1, colors, we label the vertices 0, 1, 2… 2p and then color edges according to the sum mod (2p+1) of the labels of their end points. If the vertices form a regular polygon, 0, 1, 2… 2p then a color class is a set of parallel edges. The boundary is a omni colored cycle. By deleting one edge we obtain a omni colored path on 2p+1 vertices (3).

In every optimal edge-coloring of Kp+1 exactly one color is absent at each vertex and all these absent colors are dissimilar. Expand the graph K2p+1 to K2p+2 thus we acquire a coloring of K2p+2 with 2p+1 color by coloring the edges confrontation to the new vertex with these absent colors. Assume we are able to color K2p+1 in such a way that

In the boundary every color appears precisely once expect one color c which does not appear and one color c1 which appears twice.
the vertex x where is absent is incident to an edge \(\{x, y\}\) of the boundary colored with \(c\). Extend this coloring to \(K_{2p+2}\). Let \(z\) be the new vertex then \(\{z, x\}\) is colored with \(c\) and by deleting the edge \(\{x, y\}\) from the boundary and adding up the edge \(\{x, z\}\) we obtain a omni colored path \(\{y, \ldots, x, z\}\).

We are now going to exhibit such a coloring of \(K_{2p+1}\) by some changes on the classical coloring defined above.

**Sub Case 1:** \(P = 0 \pmod{3}\)

The cycle definite by the vertices 1, 0, 2, 2p, 3, 2p-1, \ldots, 2p/3+1, 4p/3+2, 2. We change this coloring to 2, 1, 2, \ldots, 4p/3+2, 1. We have an obligatory coloring \(K_{2p+1}\) with \(c = 1\), \(c_1 = 2\), \(x = p+1\), \(Y = p+2\) in the new coloring.

**Example:** \(P = 6\)

The color missing at vertex 1 is now \(4p/3+2\) instead of 2. The color missing at vertex \(2p/3+1\) is now instead of \(4p/3+2\).

The cycle 1, 0, 2, 12, 3, 11, 4, 10, 5, 9, 1 is colored 1, 2, 1, 2, \ldots, 10. We change the coloring to 2, 1, 2, \ldots, 10, 1. Required coloring of \(K_{13}\) with \(c = 1\), \(c_1 = 2\), \(x = 7\), \(y = 8\).

In the new coloring, the color missing at vertex 1 is now 10 instead of 2. The color missing at vertex 5 is now 2 instead of 10.

**Sub Case 2:** \(P = 1 \pmod{3}\)

The path 1, 0, 2, 2p, 3, 2p-1, \((p+2)/3\), \((5p+4)/3\), \(p+1\) is colored 1, 2, 1, 2, \ldots, 2, 1, \((2p+4)/3\) and the color and the color 1 is missing at \(p+1\). We change this coloring to 2, 1, 2, 1, \ldots, 1, \((2p+4)/3\), 1. Now the color 1 is missing in 1 so that we have a required coloring of \(K_{p+1}\) with \(c = 1\), \(c_1 = 2\), \(x = 1\) and \(y = 0\) in the new coloring.

The color missing at vertex \(p+1\) is now \((2p+4)/3\) instead of 1. The color missing at vertex \((p+2)/3\) is now 2 instead of \((2p+4)/3\).

**Example:** \(P = 7\)

**Sub Case 3:** \(P = 2 \pmod{3}\)

Consider the sequence of vertices 1, 2, 4, \ldots, 2k, \ldots, 2q = \(p+1\), 1. Then all the edges between the vertex 0 and 1, 2, \ldots, 2k, \(p+1\) except the first one are not in the boundary.

In the classical coloring for each k the \((0, 2k)\) is colored with \(2k\), i.e. the sequence of edges \(\{0, 1\}, \{0, 2\}, \ldots\) is colored 1, 2, 4, \ldots, \(q\), and the color \(2k+1\) is missing at vertex \(2k\). Color this edge with \(2k+1\). At the vertex 0 we did nothing else than a circular permutation on the colors used for all these adjacent edges. Thus
we still have a coloring. This is a required coloring with c=1, c1=2, x=1 and y=0.

Sub Case 3.2:

If we are not in the above case, we have a sequence of diverse vertices 1, 2, 4, … 2k, … p, 2p. In this case we cannot use the identical change because the edge {0, 2p} is in the boundary.

Sub Case 3.2.1: P=5 (mod 6)

There is a path (p, p+2, p-2, p, 4, p-4 … 3 2p-1, 1) the edges of which are colored 1, 2p, 1, 2p … 1, 2p

P=5

Consider now the sequence of edges {0, 1}, (0, 2), (0, 4)… (0, 2k) … (0, p} (p, p+2), {p+2, p-2} {p-2, p+4} … (3, 2p-1), {2p-1, 1} colored 1, 2, 4…2k, p, 1, 2p, 1, 2p….1, 2p

This yields a required coloring with c=1, c1=2, x=p+1 and y=p+2

Sub Case 3.2.2: P=2, (mod 6)

We have P/2 = (mod 3) Consider the path P = u0, u1, u2, ..., u (2p-4)/3 u1 = p/2 – 3i/4 if I =0 (mod4)

\[ u1 = 3p/2+2+3(i-2)/4 \] if i=1(mod 4); u1 = 3p/2+2+3(i-2)/4 if i=2 (mod 4) u1 = p/2+3+3(i-3)/4 if i=3 (mod 4)

u0 = p/2, u1 = 3p/2, u2 = 3p/2-1, u1 = p/2+3

\[ u4 = u1 = p/2-3 \ u1 = 3p/2+5 \ (2p -4)/3 =p/2- \ (3(2p-4)/3)/4 \]

p/2- p/2+1=1

Consequently the path P=(p/2, 3p/2+2, 3p/2-1. p/2+3, p/2-3, 3p/2+5, 3p/2-4, p/2+6, p/2-6…4, 2p-2, p+3, p-1,1)

Therefore, edges of P are colored 1, p, 1, p…………1, p. Consider now the sequence of edges {0, 1}, (0, 2), (0, 4)… (0, p}, {p/2, 3p/2+2), {3p/2+2, 3p/2-1} …(p-1, 1) Colored 1, 2, 4…2k……p/2,1,p, 1, p…1, p1.

We have a required coloring with c=1, c1=2, x=p+1 and y=p+2

The edges of P are colored 1, p, 1, p………1, p. consider now the sequence of edge [0, 1]0 (0, 2),

… (0, 0/2}, {p/2, 3p/2+2}, (3p/2+2+ 3p/2-1)…. (p-1, 1) Colored 1, 2, 4… 2k ………p/2, 1, p, 1, p….1, p, p. we change these colors to 2, 4, …… 2k ………p/2, 1, p, 1….1, p, 1.

We have a required coloring with c=1, c1=2, x=p+1 and y=p+2

Case 2:
Sub case 2.1

In the complete graph K4, if we take any path P4, it does not have a classical color.

P4 € M4. Since the edges v1, v2 and v3 v4 being non-adjacent have the same color. i.e. the paths on 4 vertices does not belong to M4

Sub case 2.2

In the complete graph K6 we take any path P6 they have at least two edges are parallel. Therefore by classical coloring, they get the same color.

Therefore P6 € M6
i.e. the paths on 6 vertices does not belong to $M_6$
Thus we conclude that the paths on $n$ vertices belong to $M_n$ if and only if $n \neq 4, 6$
Hence the theorem.

**IV. CONCLUSION**

Hence some general results on elegant graphs, add the edge labeling and pseudo odd edge labeling are proved.

**REFERENCES**


