Comparative Study for $P_{1hr}$ Determination and Skin Factor Quantification during Drawdown Test Analysis in Oil Wells

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ABSTRACT

In conducting and analyzing tests, several parameters are recorded and estimated with each representing and detecting the nature of well/reservoir systems during and after the test periods. There are different ways of analyzing tests. This could range from single to multi-rate tests. In carrying or conducting tests when the well is put on stream could be attributed to drawdown test. The reservoir is located at some depths in the formation. Therefore, to encounter the reservoir, a well which serves as a means of accessing it is drilled. Drilling to the reservoir makes no meaning if there is no diagnosis to reveal the attributes of the well/reservoir systems. Also, having created a path, it will be easy to run for pressure recording tools/sensors into the well. The captured data could be analyzed and used for reservoir characterization. Here, drawdown test data are used to fit a model that could estimate pressure at one (1) hour and gradient taken on a cycle. This model was developed to knock off the statement that says interpolation should be used in determining pressure at one (1) hour when analyzing tests. The results of our comparison show proximity. This was achieved by introducing the modulus of the shifting term or modulus of the slope. Also, the developed equation can be turned around to determine the gradient which may not be easy to read from the graph in some cases.

INTRODUCTION

Ideally, no organization embarks on a business that does not yield profit. As we all know, the oil industry business is a capital intensive one. With this in mind, it will be meaningless to drill well without extracting anything from it. After a well is drilled, it will be incumbent on the parts of the producing firm to put it on stream. To do this, a pressure drawdown must be initiated. Also, to know the performance of the well/reservoir, tests are carried out. By putting the well on stream, recordings are made. Some of the parameters recorded and tabulated are pressure, time, temperature and so on. When all these data are captured, analyses are done in other to know the real nature/potentials of the reservoirs and wells. At the time the well is put on stream, the wellbore pressure drops steadily and the fluids closer to the well expand and migrate towards the region of least pressure. Several factors tend to hamper the fluid movements. According to Schlumberger (2006), these factors are: friction against the wall of the pore spaces and fluid’s own inertia and resistance force (viscosity). The migration of this fluid initiates a pressure drop that forces the nearby fluid to flow towards the well. This lasts until the drop in pressure which was initially initiated at the beginning is distributed in the entire reservoir. The processes that took place can be modeled with the diffusivity equation (i.e.; Fluid Flow Equation).

MODELING OF FLOW EQUATION

To analyze test with a level or degree of confidence, flow equation which is expressed in cylindrical coordinate is used. Solving this equation requires that the initial and boundary conditions be concatenated to make the model traceable. Considering a well situated in a porous system that looks infinite, the fluid flow equation is written as:
\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{\phi u c_r}{k} \frac{\partial p}{\partial t} \quad (1)
\]

Subject to the following conditions:

i. **Initial Condition**: which describes a situation in the reservoir where the pressure is the same as the initial.
   Mathematically, the initial condition is stated as:
   \[
   p(r, 0) = p_i \quad (2)
   \]

ii. **Outer Boundary Condition (OBC)**: Which states that, at infinity the pressure is equal to the initial pressure.
   The mathematical formulation describing this is:
   \[
   \lim_{r \to \infty} P(r, t) = P_i \quad (3)
   \]

iii. **Inner Boundary Condition (IBC)** specifies a constant rate production from the beginning.
   \[
   r \frac{\partial p}{\partial r} \bigg|_{r_w} = \frac{q \mu}{2 \pi k h} \quad (4)
   \]

**Figure 1: Schematic for Finite Wellbore Solution**

Or for a vanishing or diminishing wellbore (Figure 2)
   \[
   r \frac{\partial p}{\partial r} \bigg|_{r_w} \to 0 = \frac{q \mu}{2 \pi k h}
   \]

**Figure 2: Schematic of Line Source Solution**

**SOLUTIONS TO FLUID FLOW EQUATION OF DIFFUSIVITY EQUATION**

Several solutions abound in handling fluid flow equation. Some of which come from the steady state solution which assumes that pressure change does not vary with time (i.e.; \(\partial p/\partial t = 0\)). The condition is obtained in a reservoir that is passing through depletion if there is influx from an adjoining aquifer or injection into the well to supplement the reservoir energy and preclude the reservoir pressure from declining (Onyekonwu, 1997). For this kind of condition, the boundaries must be open to flow, though the entire boundary must not be open.

Another solution is the pseudo steady state solutions (PSS) which is attributed to the system where pressure decline with respect to time is a constant \(\partial p/\partial t = \text{Constant}\). The constant must not be zero but relates the volume to which the well drains (Onyekonwu). Solving PSS unveils the equation that forms the basis or guide for reservoir limit test (Onyekonwu, 1997). To make PSS feasible, there are conditions that must be attained. One of which could be that: the outer boundary of the reservoir must be closed to flow (influx); another is that the well must be draining from a bounded drainage volume because of the time of production; and finally it must also take place at late time. The state that is most paramount to the reservoir engineer and well test analyst is the **transient state**. It is an important state...
because the most useful parameters of the reservoir are derived from it (Onyekonwu, 1997). In giving a solution to this state, it is assumed that the rate of pressure change is neither constant nor zero. This character is exhibited in all systems whenever perturbation is initiated. It is a middle time region (MTR) effect and denotes a system where the perturbation initiated has not felt the drainage boundary (Figure 3).

Figure 3: Infinite Acting Radial flow (IARF)


Considering the line source solution can be obtained, equation (1) yields

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial P}{\partial r} \right) = \frac{\phi \mu c_i \partial p}{r \partial t}$$

Subject to:

$$P(r,0) = P_i$$
$$\lim_{r \to \infty} P(r,t) = P_i$$
$$\lim_{r \to 0} r \frac{\partial P}{\partial r} = -\frac{q \mu}{2 \pi kh}$$

Integrating equation (5) and apply the Boltzmann transformation. The solution to equation (1) becomes

$$P_i - P = \frac{q \mu}{4 \pi kh} E_i \left\{ -\frac{\phi \mu c_i r^2}{4 kt} \right\}$$

Equation (7) is one of the most powerful equations in well test from which all others emanate.

MATHEMATICAL METHODS OF TEST

ANALYSIS

One thing is to derive the governing flow equations and another is knowing when and when to use them. Let us first give the transient state solution in dimensionless form and another is to generalize the solution.

Starting from equation (7) and recalling that in dimensionless form, we have that:

$$P_D = \frac{2 \pi kh}{q \mu} \left[ P_i - P \right]$$

Substituting equation (7) into equation (8) then;

$$P_D = \frac{2 \pi kh}{q \mu} \left[ \frac{q \mu}{2 \pi kh} E \left[ -\frac{\phi \mu c_i r^2}{4 kt} \right] \right] = -\frac{1}{2} E_i \left[ -\frac{\phi \mu c_i r^2}{4 kt} \right]$$

But from Darcy’s units,

$$r = r_D r_w$$

and

$$t_D = \frac{kt}{\phi \mu c_i r_w^2}$$

Therefore, equation (9) becomes

$$P_D = -\frac{1}{2} E_i \left[ -\frac{\phi \mu c_i (r_D r_w)^2}{4 kt} \right] = -\frac{1}{2} E_i \left[ -\frac{r_D^2}{4 t_D} \right]$$

Onyekonwu (1997) stated that for purposes, $\frac{t_D}{r_D^2} > 5$, log approximation will hold.

Hence, the solution to Equation (12) after introducing the Euler constant ($\gamma$) becomes:

$$P_D = -\frac{1}{2} E_i \left[ -\frac{\gamma r_D^2}{4 t_D} \right]$$

By inverting Equation (13), introducing a negative sign(-ve) and noting that Euler Constant ($\gamma$) has the value of 1.781. Equation (13) degenerates to (7)
\[ P_D = \frac{1}{2} \left[ \ln \frac{4tt_D}{r_D^2} + 0.80907 \right] \]

or

\[ P_D = 1.1515 \left[ \log \frac{t_D}{r_D^2} + 0.351311 \right] \]  (14)

But at the well \( r_D^2 = 1 \); this is from that fact that \( r_D = \frac{r}{r_w} \). Equation (14) degenerates to:

\[ P_D = \frac{1}{2} \left[ \ln \frac{4tt_D}{r_w^2} + 0.80907 \right] \]

or

\[ P_D = 1.1515 \left[ \log t_D + 0.351311 \right] \]  (15)

Now, it is important to derive the equation that plays a significant role in analyzing tests during infinite acting state at the well.

From \( P_i - P_w = \frac{141.2qB\mu}{kh} \left[ P_D + S \right] \) (16)

Substituting the first part of Equation (15) into Equation (16), we have that:

\[ P_i - P_w = \frac{141.2qB\mu}{kh} \left[ \frac{1}{2} \left[ \ln \frac{4tt_D}{r_w^2} + 0.80907 \right] + S \right] \]  (17)

But dimensionless time in Oilfield Units is expressed as:

\[ t_D = \frac{0.000264}{\phi c r_w^2} kt \]  (18)

Introducing Equation (18) into Equation (17), we have that:

\[ P_i - P_w = \frac{141.2qB\mu}{kh} \left[ \frac{1}{2} \ln \left( \frac{0.000264k}{\phi c r_w^2} + 0.80907 \right) + S \right] \]  (19)

On simplifying Equation (19), we have that:

\[ P_i - P_w = \frac{141.2qB\mu}{kh} \left[ \frac{1}{2} \ln(0.000264k) + \frac{0.80907}{0.000264k} \right] \]

\[ + 2.302 \times \frac{1}{2} \log 1.1513 \]  (20)

Finally, the flow equation during transient will yield

\[ P_i - P_w = \frac{162.6qB\mu}{kh} \left[ \log t + \log \frac{k}{\phi c r_w^2} - 3.227 \right] \]  (21)

Equation (21) is the basis for estimating effect of pressure at the well if the system is in infinite acting state. There is also a useful equation that could be used for estimating pressure at a particular time and distance in the reservoir when the system is in infinite acting state. This can also be derived from Equation (7). To do this, assume that log approximation held and re-write Equation (7) with respect to distance (r) and time (t) as:

\[ P(r, t) = P_i - \frac{q\mu}{4\pi kh} \ln \left( \frac{r^2 c r_w^2}{4k t} \right) \]  (22)

Changing to natural logarithm and inverting Equation (22), then:

\[ P(r, t) = P_i - \frac{q\mu}{4\pi kh} * 2.302 \times \log \left( \frac{k t}{\phi c r_w^2} \right) \]  (23)

By expanding Equation (23), we have that:

\[ P(r,t) = P_i - \frac{1}{2} \frac{q\mu}{2\pi kh} \left( \log + \log \frac{k}{\phi c r_w^2} + 3.227 \right) \]

Finally,\[ P(r,t) = P_i - \frac{162.6q\mu}{kh} \left( \log t + \log \frac{k}{\phi c r_w^2} - 3.227 \right) \]  (24)

In other to state the condition of the well, it is necessary to make estimate of skin factor. This is made feasible by making skin (S) the subject from Equation (25) as

\[ S = 1.1513 \left( \frac{P_i - P_{1hr}}{m} - \log \frac{k}{\phi c r_w^2} + 3.227 \right) \]  (25)

Equation (25) is the fundamental equation for determining the condition of the well. That is to say that: If S = 0, there is no damage or improvement. For S negative, the well is stimulated and finally for S = Positive, the well is impaired.

From the foregoing, the value of \( P_{1hr} \) in Equation (25) previously could be obtained from semi-log plot of pressure against time without any mathematical formulation. The reason for this is that at one (1) hour, the pressure may still be affected by the wellbore storage. This informed the generation of a model that could take care of the situation whether or not the pressure at one (1) hour is affected by wellbore storage during drawdown test. But for us to
determine the $P_{thr}$, the conventional way of semi-log plot will be made and the slope $f(p)$ determined. Also, there are cases where determining the slope looks impossible, this can be from the case where the plot looks so horizontal or flat as different values will emerge. A quick way of determining the slope $(m)$ is to make it the subject from $P_{thr}$ equation as written in equations 26 and 27 respectively.

The new models are:

$$P_{thr}=P_{wf}^{n+1}+\left[ P_{wf}^{n+1}-P_{wf}^{n+1}\right] \frac{\log_{w}^{n}-\log_{w}^{n+1}}{\log_{w}^{n+1}-\log_{w}^{n+1}}+f(p)$$

But when the straight line from the semi-log graph looks flat as to make the two data points difficult to read, we are expected to read off the $P_{thr}$ from the graph and then substitute the value into Equation 27 to evaluate the slope $f(p)$ as:

$$f(p)=\frac{2303 \left\{ P_{thr}-\left[ P_{wf}^{n+1}-P_{wf}^{n+1}\right] \frac{\log_{w}^{n}-\log_{w}^{n+1}}{\log_{w}^{n+1}-\log_{w}^{n+1}} \right\} }{2303}$$

The second term on the right hand side of equation (26) is what we called the modulus of the correction term. The meaning of this that is the value of $f(p)$ that will be used in the skin equation must be positive. This correction or shifting term with the data available had shown satisfactory results when compared with that read directly from the plot. The actual procedure to use Equation (26) is as shown in Table 1.

Table 1. Measured Drawdown Pressure Values and Interpolating Points

<table>
<thead>
<tr>
<th>Time (hrs)</th>
<th>Flowing Pressure (Psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{n+1}$</td>
<td>$P_{wf}^{n+1}$</td>
</tr>
<tr>
<td>$t_{n}$=1.0 (always)</td>
<td>Not used</td>
</tr>
<tr>
<td>$t_{n+1}$</td>
<td>$P_{wf}^{n+1}$</td>
</tr>
</tbody>
</table>

The skin factor can be quantified using the new model as:

$$s = 1.1513 \left\{ P_{i} - P_{thr} - \log\left( \frac{k \phi \mu c}{r_{w}^{2}} \right) + 3.23 \right\}$$

PRESSURE DERIVATIVE CONCEPTS

Pressure derivative is used basically for identifying phases. It also eliminates the tedious of making several plots before analyzing test. Furthermore, it shows exactly where a particular phase ceases to exist or presents cases where phase transition occurs. This is one of the most powerful well testing analysis techniques used to distinguish phases a well or reservoir passes. The concept was introduced by [4]. It accounts for the fact that the pressure data when conducting a test is recorded at discrete times. Hence, pressure derivative is obtained numerically. Ref [13] developed an algorithm which can be used in obtaining the derivative. According to ref [11], one of the required algorithms is

$$\frac{\partial P}{\partial \ln t} = f\left( \ln \frac{t}{t_{k}} \right) = \frac{\ln \left( \frac{t_{i}}{t_{i+k}} \right) \frac{\partial P}{\partial \ln t} + \ln \left( \frac{t_{i+k}}{t_{i}} \right) \frac{\partial P}{\partial \ln t_{k}}}{\ln \left( \frac{t_{i}}{t_{i+k}} \right) \frac{\partial P}{\partial \ln t} - \ln \left( \frac{t_{i+k}}{t_{i}} \right) \frac{\partial P}{\partial \ln t_{k}}}$$

From Figure 4, there is wellbore storage phase, the transition and the infinite acting radial flow phases and the boundary effect.
Figure 4: Drawdown Analysis using Pressure Derivative Plot

GENERAL PROCEDURE FOR DRAWDOWN TEST ANALYSIS

i. Recorded time values in hours, minutes and seconds are properly tabulated. Though in our own analysis and using field units, the times that are recorded in different units are all converted to hours and pressure in pounds per square inch (psi).

ii. Again, the pressure at time zero is called the initial pressure and must be kept constant, and then flowing pressures are subtracted from it (i.e., $P_i - P_{wf} = \Delta P$).

iii. Then, a graph the change in pressure ($\Delta P$, psi) against flowing time (t) on a log-log paper is made. This means that both axes will be in log scales. Here, the table containing the values of times and pressures are plotted direct and not to converted using hand calculator.

iv. Then locate the data strongly influenced by the wellbore storage effect on a unit slope line.

v. Quantify the wellbore storage effect using Equation (30)

$$C = \frac{qB_o}{24} \left\{ \frac{t}{\Delta P} \right\}_{USL} \quad (30)$$

vi. A step further is to locate the data points that are not strongly influenced by the wellbore storage effect. Here, the gentle slope rule approach is used to get rid of where the wellbore storage dampens completely.

vii. Then a semi log graph of flowing pressure versus time on the log scale is plotted.

viii. Beyond the point where the wellbore storage effect dampens completely, a straight line is drawn to determine the slope on a unit cycle.

ix. The value of the slope is substituted in equations (31) and (32) to determine the permeability ($k$) and skin ($s$) as:

$$k = \frac{162.6qB\mu}{mh} \quad (31)$$

And

$$s = 1.1513 \left( \frac{P_{iw} - P_{w(t_2)}}{m} - \log \left( \frac{k}{\phi \mu \gamma_w} \right) + 3.23 \right) \quad (32)$$

Figure 5: Log-Log Plot for Drawdown test

PRESENTATION OF RESULTS FROM PLOTS AND NEW MODEL

All the procedures enumerated for well test analysis using drawdown test will be displayed alongside the results. From the data available, it was found that the
value of P1hr from the plot and that from the new model compares favourably. Also observed was the fact that any time there is much discrepancy between the P1hr from the plot and that from the model, the recorded pressure value may be in error and should be reviewed. This reason came from the fact that the people who conducted the test failed to report all that transpired to avoid being victimized by their employers.

Table 2: Pressure-Time Data for Drawdown Test Analysis (Onyekonwu, 1997)

<table>
<thead>
<tr>
<th>Time(hrs)</th>
<th>Pwf (Psi)</th>
<th>ΔP (Psi)</th>
<th>dp'</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>3126.553</td>
<td>57.21</td>
<td>3.067042</td>
</tr>
<tr>
<td>0.0888</td>
<td>3125.731</td>
<td>58.032</td>
<td>1.590262</td>
</tr>
<tr>
<td>0.1104</td>
<td>3125.446</td>
<td>58.317</td>
<td>1.214207</td>
</tr>
<tr>
<td>0.1344</td>
<td>3125.224</td>
<td>58.539</td>
<td>1.101032</td>
</tr>
<tr>
<td>0.1776</td>
<td>3124.928</td>
<td>58.835</td>
<td>1.054834</td>
</tr>
<tr>
<td>0.2496</td>
<td>3124.572</td>
<td>59.191</td>
<td>1.039147</td>
</tr>
<tr>
<td>0.3744</td>
<td>3124.154</td>
<td>59.609</td>
<td>1.020727</td>
</tr>
<tr>
<td>0.5376</td>
<td>3123.788</td>
<td>59.975</td>
<td>1.003063</td>
</tr>
<tr>
<td>0.7776</td>
<td>3123.421</td>
<td>60.342</td>
<td>0.986874</td>
</tr>
<tr>
<td>1.0176</td>
<td>3123.157</td>
<td>60.606</td>
<td>0.976614</td>
</tr>
<tr>
<td>1.2576</td>
<td>3122.951</td>
<td>60.812</td>
<td>0.973092</td>
</tr>
<tr>
<td>1.4976</td>
<td>3122.781</td>
<td>60.982</td>
<td>0.967234</td>
</tr>
<tr>
<td>1.7376</td>
<td>3122.638</td>
<td>61.125</td>
<td>0.964243</td>
</tr>
</tbody>
</table>

From Table 2, the following sets of plots are made and results of three tested wells presented.

![Figure 6: Plot Showing Derivative of Drawdown data](image)

![Figure 7: Plot Show Flowing Pressure versus log of Time](image)

Table 3: Results of P1hr and Skin from Plots and Models

<table>
<thead>
<tr>
<th>DATA for Different Wells</th>
<th>PLOT P1hr (Psi)</th>
<th>Skin Factor</th>
<th>Model P1hr (Psi)</th>
<th>Skin Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3123.157</td>
<td>25</td>
<td>3124.132</td>
<td>24.42</td>
</tr>
<tr>
<td>B</td>
<td>5600</td>
<td>-3.56</td>
<td>5618.23</td>
<td>-4.12</td>
</tr>
<tr>
<td>C</td>
<td>4939.64</td>
<td>25</td>
<td>4940.38</td>
<td>24.42</td>
</tr>
</tbody>
</table>
CONCLUSION

In conclusion, this research only did a comparative study of the values of $P_{1hr}$ and skin from plots and model. In addition, it maintained the conventional way or straight line method of well test analysis. From the foregoing, this research presents the following:

i. Fitting a new model that can predict $P_{1hr}$ and quantify skin factor without much variation.

ii. The model can also be used to evaluate the slope $f(p)$ from the semi-log plot if the plot look very flat as to make points reading vague.

iii. This model does not overestimate the value of skin mostly in analyzing drawdown test.

iv. The data which made this model feasible were obtained from the Niger Delta Oil wells

v. Finally, this work hereby state that whenever the $P_{1hr}$ from the plot varies significantly with that from the model, the test should be reviewed and data quality check initiated. The reason for this variation may have arisen from the fact that some incidents that occurred during the test were not reported.

Future Work

i. Also fit a universally accepted $P_{1hr}$ model for analyzing gas well test during Drawdown and Buildup Test

REFERENCES


NOMENCLATURE

$f(p)$ = slope from semi-log plot, psi/cycle
$\phi$ = porosity, fraction or percentage
$k_0$ = permeability, mD
$\mu$ = viscosity, cP
$ct$ = total compressibility, psi$^{-1}$
$r_w$ = wellbore radius, ft
$h$ = pay thickness, ft
$P_{1hr}$ = the value of pressure at shut-in time of one hour, psi
$\left|f(p)\right|$ = Modulus of the slope, psi/cycle
2.303 = conversion factor
$P_i$ = Initial Pressure (psi)
$r_w$ = wellbore radius (ft.)
$C_s$ = wellbore storage constant, rb/psi
$q$ = production rate, STB/D
$B_o$ = Oil Formation Volume Factor (FVF), rb/psi
$t_{n,1}$ = Value of flowing time below one (1) hour, hrs
$t_{n}$ = Value flowing time at which pressure is to modified (one (1) always), hrs
$t_{n+1}$ = Value of flowing time above one (1) hour, hrs
$P_{wf}^{n-1}$ = flowing pressure corresponding to flowing time below one (1) hour, psi
$P_{wf}^{n+1}$ = flowing pressure corresponding to flowing time above one (1) hour, psi

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