Application of Fuzzy Soft Set in Selection Decision Making Problem

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Abstract: In our daily life we often face some problems in which the right decision making is highly essential. But in most of the cases we become confused about the right solution. To obtain the best feasible solution of these problems we have to consider various parameters relating to the solution. For this we can use the best mathematical tool called Fuzzy soft set theory. In this paper we select a burning problem for the parents and successfully applied the fs-aggregation algorithm in decision making for selecting a suitable bride by the family.

Keywords: Fuzzy set, Fuzzy Soft set, fs-aggregation

1. INTRODUCTION:

In many complicated problems arising in the fields of engineering, social science, economics, medical science etc involving uncertainties. Classical methods are found to be inadequate in recent times. Molodtsov [13] pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. In 1999 he initiated the novel concept of Soft Set as a new mathematical tool for dealing with uncertainties. Soft Set Theory, initiated by Molodtsov [13], is free of the difficulties present in these theories.

Molodtsov applied this theory to several directions [13], [14], [15], and then formulated the notions of soft number, soft derivative, soft integral, etc. in [16]. The soft set theory has been applied to many different fields with great success. Maji et al. [11] worked on theoretical study of soft sets in detail, and [10] presented an application of soft set in the decision making problem using the reduction of rough sets [18]. Chen et al. [5] proposed parameterization reduction of soft sets, and then Kong et al. [7] presented the normal parameterization reduction of soft sets.


Maji et al. [9] presented the concept of the fuzzy soft sets (fs-sets) by embedding the ideas of fuzzy sets [24]. By using this definition of fs-sets many interesting applications of soft set theory have been expanded by some researchers. Roy and Maji [20] gave some applications of fs-sets. Som [21] defined soft relation and fuzzy soft relation on the theory of soft sets. Krishna Gogoi et al [27] applied fuzzy soft set and Bhardwaj et al. [28] used Reduce soft set for real life decision making problems.

The operations of the fs-sets and soft sets defined by Maji et al. [9],[11] are used in all the works mentioned above. But, Chen et al. [5], Pei and Miao [19], Kong et al. [6] and Ali et al. [1] pointed out that these works have some weak points. Therefore, to develop the theory, Cagman and Enginoğlu [2] redefined operations of the soft sets which are more functional for improving several new results. By using these new operations, Cagman and Enginoğlu [3] presented a soft matrix theory. Cagman et al. [4] defined a fuzzy parameterized soft set theory and its decision making method.

In this paper author is defining fuzzy soft set and applying fuzzy soft set aggregation method to solve decision making problems. The fs-aggregation algorithm is well defined by Cagman et al. [26] for decision making. We finally give an example which shows that the method can be successfully applied to many problems containing uncertainties.
II. PRELIMINARIES:

In this section, we present the basic definitions of soft set theory [13] and fuzzy set theory [24] that are useful for subsequent discussions. Throughout this work, U refers to an initial universe, E is a set of parameters, P (U) is the power set of U, and A ⊆ E.

Definition 2.1. A soft set F_A over U is a set defined by a function f_A representing a mapping

\[ f_A : E \rightarrow P(U) \]

such that \( f_A(x) = \emptyset; \) if \( x \not\in A \)

Here, \( f_A \) is called approximate function of the soft set \( F_A \), and the value \( f_A(x) \) is a set called \( x \)-element of the soft set for all \( x \in E \). It is worth noting that the sets \( f_A(x) \) may be arbitrary, empty, or have nonempty intersection. Thus a soft set over U can be represented by the set of ordered pairs

\[ F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\} \]

the set of all soft sets over U will be denoted by \( S(U) \).

Example 2.1. Let \( U = \{u_1, u_2, u_3, u_4, u_5\} \) be a universe and \( E = \{x_1, x_2, x_3, x_4\} \) be a set of parameters. If \( A = \{x_1, x_2, x_3, x_4\} \subseteq E \), \( f_A(x_1) = \{u_2, u_4\} \), \( f_A(x_2) = U \) and \( f_A(x_3) = \{u_1, u_3, u_5\} \), then the soft set \( F_A \) is written by

\[ F_A = \{(x_1, \{u_2, u_4\}), (x_2, U), (x_3, \{u_1, u_3, u_5\})\} \]

Definition 2.2. Let \( U \) be a universe. A fuzzy set \( X \) over \( U \) is a set defined by a function \( \mu_X \) representing a mapping

\[ \mu_X : U \rightarrow [0, 1] \]

\( \mu_X \) is called the membership function of \( X \), and the value \( \mu_X(u) \) is called the grade of membership of \( u \in U \). The value represents the degree of \( u \) belonging to the fuzzy set \( X \). Thus, a fuzzy set \( X \) over \( U \) can be represented as follows:

\[ X = \{\mu_X(u)/u, u \in U, \mu_X(u) \in [0, 1]\} \]

The set of all the fuzzy sets over \( U \) will be denoted by \( F(U) \).

In the soft sets, the parameter sets and the approximate functions are crisp. But in the fuzzy sets, while the parameters sets are crisp, the approximate functions are fuzzy subsets of \( U \). From now on, we will use \( \Gamma_A, \Gamma_B, \Gamma_C, \ldots \) etc., for \( F \)-sets and \( \gamma_A, \gamma_B, \gamma_C, \ldots \) etc., for their fuzzy approximate functions, respectively.

Definition 2.3. An \( F \)-set \( \Gamma_A \) over \( U \) is a set defined by a function \( \gamma_A \) representing a mapping

\[ \gamma_A : E \rightarrow F(U) \]

such that \( \gamma_A(x) = \emptyset; \) if \( x \not\in A \).

Here, \( \gamma_A \) is called fuzzy approximate function of the \( F \)-set \( \Gamma_A \), and the value \( \gamma_A(x) \) is a set called \( x \)-element of the \( F \)-set for all \( x \in E \). Thus, an \( F \)-set \( \Gamma_A \) over \( U \) can be represented by the set of ordered pairs

\[ \Gamma_A = \{(x, \gamma_A(x)) : x \in E; \gamma_A(x) \in F(U)\} \]

Note that the set of all \( F \)-sets over \( U \) will be denoted by \( F(U) \).

Example 2.2. Let \( U = \{u_1, u_2, u_3, u_4, u_5\} \) be a universal set and \( E = \{x_1, x_2, x_3, x_4\} \) be a set of parameters. If \( A = \{x_1, x_2, x_3, x_4\} \subseteq E \), \( \gamma(x_1) = \{0.9/u_2, 0.5/u_4\} \), \( \gamma(x_2) = U \), and \( \gamma(x_3) = \{0.2/u_1, 0.4/u_3, 0.8/u_5\} \) then the soft set \( F_A \) is written by

\[ F_A = \{(x_1, \{0.9/u_2, 0.5/u_4\}), (x_2, U), (x_3, \{0.2/u_1, 0.4/u_3, 0.8/u_5\})\} \]

Definition 2.4. Let \( \Gamma_A \in FS(U) \). If \( \gamma_A(x) = \emptyset; \) for all \( x \in E \), then \( \Gamma_A \) is called an empty \( F \)-set, denoted by \( \Gamma \Phi \).

Definition 2.5. Let \( \Gamma_A \in FS(U) \). If \( \gamma_A(x) = U; \) for all \( x \in A \), then \( \Gamma_A \) is called \( A \)-universal \( F \)-set, denoted by \( \Gamma_A \).

If \( A = E \), then the \( A \)-universal \( F \)-set is called universal \( F \)-set, denoted by \( \Gamma_E \).

Example 2.3. Assume that \( U = \{u_1, u_2, u_3, u_4, u_5\} \) is a universal set and \( E = \{x_1, x_2, x_3, x_4\} \) is a set of all parameters. If \( A = \{x_1, x_2, x_3, x_4\} \subseteq E \), \( \gamma(x_1) = \{0.5/u_2, 0.9/u_4\} \), \( \gamma(x_2) = \emptyset \); and \( \gamma(x_i) = U \), then the \( F \)-set \( \Gamma_A \) is written by

\[ \Gamma_A = \{(x_1, \{0.5/u_2, 0.9/u_4\}), (x_2, U)\} \]

If \( B = \{x_1, x_2\} \), and \( \gamma_B(x_1) = \emptyset \), \( \gamma_B(x_2) = \emptyset \), then the \( F \)-set \( \Gamma_B \) is an empty \( F \)-set, i.e., \( \Gamma_B = \Gamma \Phi \).

If \( C = \{x_1, x_2\} \), \( \gamma_C(x_1) = U \), and \( \gamma_C(x_2) = U \), then the \( F \)-set \( \Gamma_C \) is a \( C \)-universal \( F \)-set, i.e., \( \Gamma_C = \Gamma_E \).

If \( D = E \), and \( \gamma_D(x_i) = U \) for all \( x_i \in E \), where \( i = 1,2,3,4 \), then the \( F \)-set \( \Gamma_D \) is a universal \( F \)-set, i.e., \( \Gamma_D = \Gamma_E \).

III. FS-AGGREGATION ALGORITHM:

We define an \( F \)-aggregation operator that produces an aggregate fuzzy set from an \( F \)-set and its cardinal set. The approximate functions of an \( F \)-set are fuzzy. An \( F \)-aggregation operator on the fuzzy sets is an operation by which several approximate functions of an \( F \)-set are combined to produce a single fuzzy set which is the aggregate fuzzy set of the \( F \)-set. Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set.

Therefore, we can make a decision by the following algorithm.

Step 1: Construct an \( F \)-set \( \Gamma_A \) over \( U \).

Step 2: Find the cardinal set \( c(\Gamma_A) \) of \( \Gamma_A \).

Step 3: Find the aggregate fuzzy set \( \Gamma(\Gamma_A) \).

Step 4: Find the best alternative from this set that has the largest member-ship grade by \( \max \Gamma(\Gamma_A)(u) \).

A. Step 1

Let \( \Gamma_A \in FS(U) \). Assume that \( U = \{u_1; u_2; \ldots; u_m\} \), \( E = \{x_1; x_2; \ldots; x_n\} \) and \( A \subseteq E \), then the \( \Gamma_A \) can be presented by the following table.
\[ \Gamma \] is the membership function of \( \Gamma \).

If \( \mu_{\gamma \Delta}^{\gamma}(x_j)(u_i) \) for \( i=1,2,\ldots,m \) and \( j=1,2,\ldots,n \) then the fs-set \( \Gamma \) is uniquely characterized by the matrix

\[
[a_{ij}]_{mn} = \begin{bmatrix}
a_{11} & a_{12} & \ldots & a_{1n} \\
a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
am_1 & am_2 & \ldots & am_n
\end{bmatrix}
\]

This matrix is called an \( m \times n \) fs-matrix of the fs-set \( \Gamma \) over \( U \).

### B. Step 2

Let \( \Gamma \in \text{FS}(U) \) then the cardinal set \( \Gamma \) denoted by \( c\Gamma \) and defined by \( c\Gamma = \{ \mu_{c\Gamma}(x) / x : x \in E \} \) is a fuzzy set over \( E \). The membership function \( \mu_{c\Gamma} \) of \( c\Gamma \) is defined by

\[
\mu_{c\Gamma} : E \rightarrow [0,1], \quad \mu_{c\Gamma}(x) = \frac{\lambda_{A(x)}}{|U|}
\]

where \( |U| \) is the cardinality of universe \( U \), and \( \lambda_A(x) \) is the scalar cardinality of fuzzy set \( \gamma \Delta(x) \).

The set of all cardinal sets of the fs-sets over \( U \) will be denoted by \( c\text{FS}(U) \subseteq \text{F}(E) \).

Now let \( \Gamma \in \text{FS}(U) \) and \( c\Gamma \in c\text{FS}(U) \). Assume that \( E = \{ x_1, x_2, \ldots, x_n \} \) and \( A \subseteq E \), then \( c\Gamma \) can be presented by the following table

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( X_1 )</th>
<th>( X_2 )</th>
<th>( \ldots )</th>
<th>( X_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>( \mu_{\gamma \Delta}(x_1)(u_1) )</td>
<td>( \mu_{\gamma \Delta}(x_2)(u_1) )</td>
<td>( \mu_{\gamma \Delta}(x_n)(u_1) )</td>
<td></td>
</tr>
<tr>
<td>( u_2 )</td>
<td>( \mu_{\gamma \Delta}(x_1)(u_2) )</td>
<td>( \mu_{\gamma \Delta}(x_2)(u_2) )</td>
<td>( \mu_{\gamma \Delta}(x_n)(u_2) )</td>
<td></td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td></td>
</tr>
<tr>
<td>( u_m )</td>
<td>( \mu_{\gamma \Delta}(x_1)(u_m) )</td>
<td>( \mu_{\gamma \Delta}(x_2)(u_m) )</td>
<td>( \mu_{\gamma \Delta}(x_n)(u_m) )</td>
<td></td>
</tr>
</tbody>
</table>

If \( a_{ij} = \mu_{c\Gamma}(x_j)(u_i) \) for \( i=1,2,\ldots,m \) and \( j=1,2,\ldots,n \) then the cardinal set \( c\Gamma \) is uniquely characterized by the matrix

\[
[a_{ij}]_{mn} = \begin{bmatrix}
a_{11} & a_{12} & \ldots & a_{1n} \\
a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
am_1 & am_2 & \ldots & am_n
\end{bmatrix}
\]

Which is called the cardinal matrix of the cardinal set \( c\Gamma \) over \( E \).

### C. Step 3

Let \( \Gamma \in \text{FS}(U) \) and \( c\Gamma \in c\text{FS}(U) \). Then fs-aggregation operator, denoted by \( \text{FSagg} \), is defined by

\[
\text{FSagg} : c\text{FS}(U) \times \text{FS}(U) \rightarrow \text{F}(U), \quad \text{FSagg}(c\Gamma, \Gamma) = \Gamma^*A
\]

Where \( \Gamma^*A = \{ \mu_{\Gamma^*A}(u) / u \in U \} \) is a fuzzy set over \( U \). \( \Gamma^*A \) is called the aggregate fuzzy set of the fs-set \( \Gamma \). The membership function \( \mu_{\Gamma^*A} \) of \( \Gamma^*A \) is denoted as follows:

\[
\mu_{\Gamma^*A}(u) = \frac{1}{|E|} \sum_{x \in E} \mu_{c\Gamma}(x)\mu_{\gamma \Delta}(x)(u)
\]

where \( |E| \) is the cardinality of \( E \).

Now assume that \( U = \{ u_1, u_2, \ldots, u_m \} \), then the \( \Gamma^*A \) can be presented by the following table

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( \mu_{\Gamma^*A} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>( \mu_{\Gamma^*A}(u_1) )</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>( \mu_{\Gamma^*A}(u_2) )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( u_m )</td>
<td>( \mu_{\Gamma^*A}(u_m) )</td>
</tr>
</tbody>
</table>

If \( a_{1i} = \mu_{\Gamma^*A}(u_j) \) for \( i=1,2,\ldots,m \) then \( \Gamma^*A \) is uniquely characterized by the matrix \( A_{1i} \)

\[
[A_{1i}]_{mx1} = \begin{bmatrix}
a_{11} & a_{12} & \ldots & a_{1n} \\
a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
am_1 & am_2 & \ldots & am_n
\end{bmatrix}
\]
Which is called the aggregate matrix of $\Gamma^*A$ over $U$.

If $M_{\Gamma A}$, $M_{c\Gamma A}$, $M_{\Gamma^*A}$ are representation matrices of $\Gamma A$, $c\Gamma A$ and $\Gamma^*A$, respectively then

$$IEI \times M_{\Gamma^*A} = M_{\Gamma A} \times \frac{1}{IEI}M_{c\Gamma A}^T$$

Then $M_{\Gamma^*A} = \frac{1}{IEI}M_{\Gamma A} \times M_{c\Gamma A}^T$,

Where $M_{c\Gamma A}^T$ is the transposition of $M_{c\Gamma A}$ and $IEI$ is the cardinality of $E$.

**D. Step 4**

Find the best alternative from this aggregate fuzzy set $\Gamma^*A$ that has the largest member-ship grade by $\text{max} \Gamma^*(u)$.

**IV. APPLICATION:**

A family wants to choose a bride groom among five girls. The family give different weight in terms of fuzzy set to the girls according to their gentry, education, beauty, job and elegance. So the five girls who form the set of alternatives, $U= \{u_1, u_2, u_3, u_4, u_5\}$ and the selection parameters of the family make a set of parameters, $E=\{x_1, x_2, x_3, x_4, x_5\}$. The parameter $x_i$ for $i=1,2,3,4,5$ stand for gentry, education, beauty, job and elegance respectively. For making a right decision for selecting the suitable bride, the fs-aggregation algorithm is applied here as follows.

*Step 1.*: Construct an fs-set $\Gamma A$ over $U$ as given in the following table

<table>
<thead>
<tr>
<th>$\Gamma A$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>0.5</td>
<td>0.7</td>
<td>0.6</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$u_2$</td>
<td>0.6</td>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>$u_3$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.7</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>$u_4$</td>
<td>0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>$u_5$</td>
<td>0.4</td>
<td>0.6</td>
<td>0.7</td>
<td>0.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Then $[a_{ij}]_{m,n}$ is called an m x n fs-matrix of the fs-set $\Gamma A$ over $U$ as given below.

$$[a_{ij}]_{m,n} = \begin{bmatrix}
0.5 & 0.7 & 0.6 & 0.8 & 0.8 \\
0.6 & 0.5 & 0.5 & 0.7 & 0.4 \\
0.4 & 0.6 & 0.7 & 0.5 & 0.7 \\
0.9 & 0.5 & 0.5 & 0.6 & 0.4 \\
0.4 & 0.6 & 0.7 & 0.5 & 0.7
\end{bmatrix}$$

Step 2: The cardinal set $c\Gamma A$ of $\Gamma A$ is computed as follows:

$$\mu_{\Gamma A}(x_1) = \frac{0.5 + 0.6 + 0.4 + 0.9 + 0.4}{5} = 0.56$$
$$\mu_{\Gamma A}(x_2) = \frac{0.7 + 0.5 + 0.6 + 0.5 + 0.6}{5} = 0.58$$
$$\mu_{\Gamma A}(x_3) = \frac{0.6 + 0.5 + 0.7 + 0.5 + 0.7}{5} = 0.60$$
$$\mu_{\Gamma A}(x_4) = \frac{0.8 + 0.7 + 0.5 + 0.6 + 0.5}{5} = 0.62$$
$$\mu_{\Gamma A}(x_5) = \frac{0.8 + 0.4 + 0.7 + 0.4 + 0.7}{5} = 0.60$$

So cardinal set $c\Gamma A = \{0.56/x_1, 0.58/x_2, 0.60/x_3, 0.62/x_4, 0.60/x_5\}$

Step 3: The aggregate fuzzy set $M_{\Gamma^*A}$ is computed as follows:

$$M_{\Gamma^*A} = \begin{bmatrix}
0.5 & 0.7 & 0.6 & 0.8 & 0.8 & 0.56 \\
0.6 & 0.5 & 0.7 & 0.4 & 0.58 \\
0.4 & 0.6 & 0.7 & 0.5 & 0.7 & 0.60 \\
0.9 & 0.5 & 0.6 & 0.4 & 0.4 & 0.62 \\
0.4 & 0.6 & 0.7 & 0.5 & 0.7 & 0.60
\end{bmatrix}$$

That means,

$$\Gamma^*A = \{0.208/u_1, 0.144/u_2, 0.174/u_3, 0.141/u_4, 0.174/u_5\}$$

Step 4: Finally the largest membership grade is chosen by $\text{max} \mu_{\Gamma^*A}(u) = 0.208$

Which means that that the bride $u_1$ has the largest membership grade, hence $u_1$ is the best suitable match among five girls.

**V. CONCLUSION:**

In the present paper we aim to give an alternative way for computation of fuzzy soft decision making problem in more precisely qualitative than the existed methods. By using fs-aggregation method, we obtain the optimum logical results in an easier and faster way. To develop the theory, in this work, we first defined fs-sets and their operations. We then presented the decision making method for the fs-set theory. Finally, we provided an example demonstrating the successfully application of this method. It may be applied to many fields with problems that contain uncertainty, and it would be beneficial to extend the proposed method to subsequent studies.
References:


