Study of Chemical Reaction on Effects on an Unsteady Mhd Heat And Mass Transfer Flow Past A Semi Infinite Vertical Porous Moving Plate In The Presence Of Viscous Dissipation

* M.N Rajashekar¹, B.Shankar Goud ²

¹Department of Mathematics, JNTUH College of Engineering Nachupally, Karimnagar -505501, TS, India.
²Department of Mathematics, JNTUH College of Engineering, Kukapatelly, Hyderabad- 085, TS, India.

Abstract: In this paper we analyse the effects of chemical reaction and inclined magnetic field on unsteady magnetohydrodynamic heat and mass transfer flow past a semi infinite moving vertical porous plate in the presence of viscous dissipation. The governing equations are solved by Galerkin finite element method. A parametric study described the influence of the various physical parameters on the temperature concentration, velocity. The results of this analysis as shown in graphically as well as on the skin friction and rate of heat and mass transfer coefficients are derived and discussed numerically.

Keywords : Magnetic field, MHD, Galerkin method, chemical reaction, viscous dissipation.

I. INTRODUCTION

The analysis of flow and heat transfer in porous medium has received much attention due to its ever increasing applications in industries and engineering technology. MD.Abdul Sattar [1] investigated free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. Y.J. Kim [2] studied unsteady convection flow of micropolar fluids past a vertical porous plate embedded in a porous medium. Heat and mass transfer along a vertical plate with variable surface tension and concentration in the presence of the magnetic field was presented by E.M.A Elbashbeshy[3]. B.Vasu et.al [4] observed radiation and mass transfer effects on Transient free convection flow of a dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. E.M.A Elbashbeshy and M.A.A Bazid [5] were studied heat transfer over an unsteady stretching surface with internal heat generation. R.Muthucumaraswamy [6] studied the interaction of thermal radiation on vertical oscillating plate with variable temperature and mass diffusion. M.S.Alam et.al [7] have analyzed the transient magnetohydrodynamic free convection heat and mass transfer flow with thermophoresis past a radiate inclined chemical reaction and temperature dependent viscosity. R.C Choudary and Arpita Jain [8] investigated an exact solution of magnetohydrodynamic convection flow past an accelerated surface embedded in porous medium. S.Siviah et.al [9] have studied finite element analysis of chemical reaction and radiation effects on isothermal vertical oscillating plate with variable mass diffusion. Thermal diffusion effect on MHD heat and Mass transfer flow past a semi infinite moving vertical porous plate with heat generation and chemical reaction was considered by Gurivireddy et.al [10].

Aim of the present paper is to investigate the effects of chemical reaction on unsteady MHD heat and mass transfer flow past a semi infinite vertical porous moving plate under the influence of a uniform transfer magnetic field in the presence of viscous dissipation with the temperature gradient. The numerical solutions of velocity, temperature and concentration are obtained by Galerkin finite element method. The effects of the various parameters on flow field velocity, temperature and concentration are analysed through graphs.

II. MATHEMATICAL FORMULATION

We consider an unsteady magnetohydrodynamic (MHD) free convective heat and mass transfer flow of a viscous, incompressible fluid along a semi – infinite vertical plate in the presence of the viscous dissipation. Let the x’ -axis is taken along the direction of the plate and y’ axis normal to it. All the fluid properties except the influence of density in the body force term are constants. We assume the free stream velocity, the plate temperature and concentration flows are increasing exponentially. The induced magnetic field is negligible because transversely applied magnetic field and magnetic.
Reynolds numbers are very small. Under the above assumptions, the governing equations are:

**Continuity equation**

\[ \frac{\partial v'}{\partial y'} = 0 \quad \text{--- (1)} \]

**Momentum equation:**

\begin{align*}
\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} &= - \frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) \\
+ g\beta'(C' - C'_\infty) - \frac{u'}{K'} &- \frac{\sigma B'_e}{\rho} (\sin^2 \alpha) u' = 0 \\
\text{--- (2)}
\end{align*}

**Energy equation:**

\[ \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{K}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + Q'(T' - T'_\infty) + \mu \left( \frac{\partial u'}{\partial y'} \right)^2 \quad \text{--- (3)} \]

**Concentration equation:**

\[ \frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = C' \frac{\partial^2 C'}{\partial y'^2} - K' \left( C' - C'_\infty \right) \quad \text{--- (4)} \]

And the appropriate boundary conditions are

\[ u' = u'_p, T' = T'_\infty + \varepsilon (T'_p - T'_\infty) e^{\varepsilon y'}, C' = C'_\infty + \varepsilon (C'_w - C'_\infty) e^{\varepsilon y} \quad \text{at } y' = 0, \]

\[ u' = u'_0 = U_0 + \varepsilon \left( 1 + e^{\varepsilon y} \right), T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \quad \text{--- (5)} \]

From the continuity of the (1), it is clear that the suction velocity normal to the plate is a function of time only and we shall take in the form of

\[ v' = v'_0 \left[ 1 + \kappa (1 - \varepsilon) \right] \quad \text{--- (6)} \]

Where \(\kappa\) is a real positive constant, \(\varepsilon\) and \(\kappa\varepsilon\) are less than unity, and \(v'_0\) is a scale of suction velocity which has non-zero positive constant. Outer surface of the boundary layer, equation (2) gives

\[ - \frac{1}{\rho} \frac{\partial u'}{\partial x'} = \frac{\partial u'_p}{\partial t'} + \frac{\sigma B'_e}{\rho} (\sin^2 \alpha) u'_\infty \quad \text{--- (7)} \]

And we introduce the non-dimensional variables, as follows

\[ u = \frac{u'}{U_0}, v = \frac{v'}{v'_0}, y = \frac{y}{v'_0}, U = \frac{U'_p}{U_0}, \]

\[ K = \frac{K'}{\nu}, \quad \text{Pr} = \frac{\rho C_p}{K}, \quad M = \frac{\sigma B'_e}{\rho v'_0}, \quad t = \frac{\nu}{v'_0}, \]

\[ Kr = \frac{\partial K\beta}{\nu^2}, \quad Gr = \frac{\partial g\beta (C'_w - C'_\infty)}{U_0 v'_0}, \quad Q = Q'K \]

\[ N = \left\{ M \sin^2 \alpha + \frac{1}{K} \right\}, \quad \text{Sc} = \frac{\partial}{\partial y} e^{C_e} = \frac{v^2}{v'_0} \quad \text{--- (8)} \]

In the view of equation (6) to (8) the governing equations reduces to

\[ \frac{\partial u}{\partial t} \left( 1 + \varepsilon A \varepsilon \right) \frac{\partial u}{\partial y} = \frac{\partial U}{\partial y} + \frac{\partial u}{\partial y} + N(U_\infty - u) \]

\[ + Gr\theta + GmC \quad \text{--- (9)} \]

\[ \frac{\partial \theta}{\partial t} \left( 1 + \varepsilon A \varepsilon \right) \frac{\partial \theta}{\partial y} = \frac{\partial \theta}{\partial y} + Q\theta + GmC \quad \text{--- (10)} \]

\[ B = Gm C + Gm C + Nu_\infty \quad \text{--- (11)} \]

The boundary and initial conditions are

\[ u = u'_0, \quad \theta = 1 + \varepsilon e^{\varepsilon y}, C = 1 + \varepsilon e^{\varepsilon y} \quad \text{at } y = 0 \]

\[ u \rightarrow U_\infty = 1 + \varepsilon e^{\varepsilon y}, \theta \rightarrow 0, C \rightarrow 0, \quad \text{at } y = 0 \]

**III SOLUTION OF THE PROBLEM**

By applying Galerkin finite element method for equation (6) over the element \(\varepsilon\), \(y_j \leq y \leq y_{j+1}\) is:

\[ \int_{y_j}^{y_{j+1}} \left[ -N^{ij} + B \frac{\partial u}{\partial y} - \frac{\partial u}{\partial x} - Nu' + R \right] dy = 0 \]

Where \(B = 1 + \varepsilon A e^{\varepsilon y}, \quad N = \left\{ M \sin^2 \alpha + \frac{1}{K} \right\} \]

\[ R = \kappa \nu e^{\varepsilon y} + (Gr \theta + GmC) C + Nu'_\infty \]

Integrating the first term in equation (15) by parts one obtains

\[ N^{ij} \left[ \frac{\partial u}{\partial y} \right]_{y_j}^{y_{j+1}} - \left( \frac{\partial u}{\partial x} \right)_{y_j}^{y_{j+1}} - Nu' + R \]

\[ \int_{y_j}^{y_{j+1}} \left[ \frac{\partial u}{\partial x} \right]_{y_j}^{y_{j+1}} = 0 \]

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Neglecting the first term in equation (12), we get:

\[
\int_{y_j}^{y_{j+1}} \left( \frac{\partial^2 \phi^{(e)}}{\partial y^2} + N^2 \left( \frac{\partial \phi^{(e)}}{\partial t} - B \frac{\partial^2 \phi^{(e)}}{\partial y^2} + N \phi^{(e)} - R \right) \right) dy = 0
\]

Let \( \mathbf{u}^{(e)} = N^{(e)} \varphi^{(e)} \) be the linear piecewise approximation solution over the element \( e \) where \( N^{(e)} = \begin{bmatrix} N_j & N_k \end{bmatrix} \), \( \varphi^{(e)} = \begin{bmatrix} u_j & u_k \end{bmatrix} \) and \( N_j = \frac{y_i - y}{y_j - y_i}, \quad N_k = \frac{y - y_i}{y_j - y_i} \)

are the basis functions. One obtains:

\[
\int_{y_j}^{y_{j+1}} \left[ N_j N'_k N'_j N'_k u_j + N'_j N_k N'_j N'_k u_k \right] dy + \int_{y_j}^{y_{j+1}} \left[ N_j N_k N_j N_k \varphi^{(e)} \right] dy - B \int_{y_j}^{y_{j+1}} \left[ N_j N'_k N'_j N'_k u_j + N'_j N_k N'_j N'_k u_k \right] dy = \int_{y_j}^{y_{j+1}} [N_j N_k N_j N_k] \varphi^{(e)} dy
\]

Simplifying we get

\[
\frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 & 0 & 0 & u_{j+1} & u_j \end{bmatrix} + \frac{1}{l^{(e)}} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \varphi^{(e)} - B \frac{1}{l^{(e)}} \begin{bmatrix} -1 & 0 & 1 & 0 & u_{j-1} & u_j \end{bmatrix} + \frac{B}{2l^{(e)}} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \varphi^{(e)} = \int_{y_j}^{y_{j+1}} [N_j N_k N_j N_k] \varphi^{(e)} dy
\]

Where \( \varphi^{(e)} \) is a function of \( u_j \) and \( u_k \). Applying the trapezoidal rule, following system of equations in Crank – Nicholson method are obtained:

\[
A_i u_{i+1} + A_2 u_{i+1} + A_4 u_{i+1} = A_4 u_i + A_4 u_i + A_4 u_{i+1} + R^T
\]

--- (15)

Where

\[ A_i = 2 - 6r + 3Brh + Nk \]

\[ A_2 = 8 + 12r + 4Nh, \quad A_4 = 8 - 12r - 4Nh \]

\[ R^T = 12Brk = 12k(nce^n + (Gr)\vartheta^n + (Gm)C_e + NU_e) \]

Now from equations (10) and (11) following equations are obtained:

\[
B_i \vartheta^{n+1} + B_2 \vartheta^{n+1} + B_4 \vartheta^{n+1} = B_4 \vartheta^n + B_4 \vartheta^n + B_4 \vartheta^{n+1} + R^T
\]

--- (16)

\[
C_1 C^{n+1} + C_1 C^{n+1} = C_4 C^n + C_4 C^n + C_4 C^n
\]

--- (17)

Here, \( B_i = 2Pr - 6r + 3Pr Brh + kPrQ \)

\[ C_2 = 8Sc + 12r + 4kScK \]

\[ C_3 = 2Sc - 6r + 3rBh.Sc + kScK \]

\[ C_4 = 8Sc - 12r - 4kScK \]

The system of equations is obtained:

\[ A_i X_i = B_i, \quad i = 1, \ldots, n \]

--- (18)

Where \( A_i \) ’s are matrix of order \( n \) and \( X_i, B_i \)’s column matrices having \( n \) components. The solutions above systems of equations are obtained by using the Thomas algorithm for velocity,
temperature and concentration. Also the numerical results are obtained by run the C-program. No significant change was found in the velocity, Temperature and concentration with the smaller values of $h, k$ then the Galerkin finite element method is stable and convergent.

The skin friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. By knowing the velocity field in the boundary layer, we can calculate the skin friction at the wall of the plate, the heat transfer coefficient in terms of Nusselt number and mass transfer coefficient in terms of Sherwood number as follows:

Skin - friction at the wall: $\tau_w = \left( \frac{\partial u}{\partial y} \right)_{y=0}$

Nusselt number: $Nu = \left( \frac{\partial \theta}{\partial y} \right)_{y=0}$

Sherwood number: $Sh = \left( \frac{\partial C}{\partial y} \right)_{y=0}$

IV. RESULTS AND DISCUSSIONS

In order to get a physical view of the problem, a set of numerical values of temperature, concentration and velocity for different values of effects of the physical parameters and shown in graphically. Figure 1, 9 and 13 shows the velocity, temperature and concentration profile for different values of dimensionless exponential index $n$. It is observed that the velocity, temperature and concentration are increases when increase in dimensionless exponential index. Figure 2 demonstrate that the effect of the velocity profile for different values of magnetic field parameter. It is observed that the velocity decreases with increase in $M$. Figure 3 illustrate the variations of the velocity for different values of Prandtl number. Clearly as $Pr$ increases the values of the velocity tends to decreases. Figure 4 depicts the velocity profile for different values of Grashof number $Gr$. It shows that an increase in $Gr$ the peak values of the velocity tends to increase. Figure 5 shows that the velocity increases with increase in permeability parameter. The velocity and concentration profiles for different values of chemical reaction parameter are described in figure 6 and 15. It is observed that the velocity and concentration decreases with increase in $Kr$. Figure 7 shows that the velocity profile for different values of Grashof number $Gc$. Clearly as $Gc$ increases the velocity tends to increase. For different values of angle of inclination the velocity profile plotted in figure 8. It is notice that the velocity decreases when increasing the angle of inclination. Figure 10 displays the temperature profile decrease when increase in Prandtl number $Pr$. Figure 11 represents the temperature profile for different values of chemical reaction parameter. It is observe that there is no effect on temperature when increasing the $Kr$ values.

![Fig. 1: Velocity profile for different values of n](image1.png)

![Fig. 2: Velocity profile for different values of M](image2.png)

![Fig. 3: Velocity profile for different values of Pr](image3.png)
Figure 12 illustrate the variations on the temperature profile for different values of Eckart number (Ec). It is notice that temperature decreases when increasing the values of Ec.

Figure 14 shows the concentration profile for different values of Schmidt number (Sc). The numerical results show that the effects of increasing the values of Sc results in a decrease in concentration. For the case of different values of Chemical reaction parameter (Kr) the concentration profiles are plotted in figure 15. It shows that the concentration decreases with increase in (Kr)

**Fig. 4: Velocity Profile for different values of Gr**

**Fig. 5: Velocity Profile for different values of K**

**Fig. 6: Velocity Profile for Different values of Kr**

**Fig. 7: Velocity profile for different values of Gc**

**Fig. 8: Velocity profile for Different values of α**

**Fig. 9: Temperature profile for different values of n**
V. CONCLUSIONS

The numerical values of the velocity, temperature and concentration flow fields are computed for different values of physical parameters like Magnetic field ($M$), Prandtl number ($Pr$), Grashof number ($Gr$), Mass Grashof number ($Gc$), heat source ($Q$), dimensionless exponential index ($n$), Schmidt number ($Sc$), Permeability of porous media ($K$), dimensional material parameter ($\alpha$), Chemical reaction parameter ($Kr$), Eckart number ($Ec$).

Solutions of the model have been studied by using Galerkin finite element method; some conclusions of our study are as below:

- Velocity increases with the increase in Magnetic field, Prandtl number, Grashof number ($Gr$), Mass Grashof number ($Gc$), heat source ($Q$), dimensionless exponential index ($n$).
number, and mass Grashof number, permeability of porous medium.

- Velocity, temperature and concentration fields increases with the increase in dimensionless exponential index.
- Velocity decreases with the increase of chemical reaction parameter and angle of inclination.
- Concentration decrease with increase in Schmidt number, chemical reaction parameter.
- Velocity decrease with increase in Prandtl number and angle of inclination.
- From Table 1 we observe that an increase in Pr, M, Sc, Kr and α results in a decrease in skin friction while an increase in \(Gr, Gc, Q, K\), and \(Ec\) results in an increase in the skin friction. An increase in Pr, M and α leads to an increase in the Nusselt number while reverse effect is noted for an increase in \(Gr, Gc, Sc, Q, K, Kr, A\) and \(Ec\).
- From table 2 an increase in \(Kr, Sc\) and \(Ec\) leads to an increase in Sherwood number while reverse effect is noted that for an increase in \(A\).

### Table 1: Numerical values of Skin – friction coefficient \(\tau_w\) and Nusselt number \((Nu)\)

<table>
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<th>M</th>
<th>Gr</th>
<th>Gc</th>
<th>Sc</th>
<th>Q</th>
<th>A</th>
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### Table 2: Numerical values of Sherwood number \((Sh)\)

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### REFERENCES


