Study of Optimum Eccentricity Angle of a Horizontal Eccentric Annulus

M. G. Mousa, Mustafa. A. El-Bouz, Mahmoud A. Elazab
Mechanical Engineering Department, Faculty of Engineering, Mansoura University,
Mansoura, Egypt

Abstract
A numerical study of mixed convection and flow inside an eccentric horizontal annulus is presented. The inner and outer cylinders rotate to generate the forced convection effect. The numerical work was carried out using ANSYS FLUENT 16. The numerical study is conducted for laminar flow mixed convection. Different eccentricity angle were studied to get the best position of eccentricity at which the best heat transfer occurs. The effect of eccentricity angle on the average Nusselt number at constant radius ratio of 2 and constant eccentricity of 0.5 is conducted. All results were performed with constant heat flux=400 W/m² at the inner cylinder and constant inlet velocity. isothersms was investigated.

Key Words : Mixed convection, Eccentricity angle, Eccentric annulus, Rotating cylinder.

1- Introduction

The heat transfer in an annulus between two horizontal concentric cylinders have attracted considerable attention because of their wide engineering applications. The study of heat transfer in an annulus has direct industrial Natural convection of nanofluid in a concentric annulus considering variable viscosity and variable thermal conductivity has been investigated by Abu-Nada [7] and Abu-Nada et al. [8]. Matin and Pop [9] studied numerically natural convection flow and heat transfer of Copper (Cu)–water nanofluid inside an eccentric horizontal annulus. Laminar mixed convection Al₂O₃-Water nanofluid flow in elliptic ducts with constant heat flux boundary condition has been simulated employing two phase mixture model by Shariat et al. [10]. Results showed that in a given Reynolds number and Richardson number, increasing solid nanoparticles volume fraction increases the Nusselt number while the skin friction factor decreases. Increasing aspect ratio in elliptic tubes reduces the local skin friction factor whereas it does not have any specified effect on the local Nusselt number. Mirmasoumi and Behzadmehr [11] studied numerically laminar mixed convection of a nanofluid consisting of water and Al₂O₃ in a horizontal applications such as heat exchangers, heat transfer in turbo machineries, indoor climate, double glazed windows, cooling of electrical and electronic components, underground electric transmission cables using pressurized gas and others.

Numerical simulation of natural convection in concentric and eccentric circular cylinder has been reported in the literatures [1-4]. Wael et al [5] studied mixed convection in an eccentric annulus filled by nanofluid. The inner and outer cylinders are kept at constant temperatures as T_b and T_c, respectively. The inner cylinder rotates to generate the forced convection effect. Omer et al [6] studied natural convection heat transfer in horizontal concentric annulus between outer cylinder and inner flat tube using nanofluid. Natural convection heat transfer in two-dimensional region Omer et al [6] studied natural convection heat transfer in horizontal concentric annulus between outer cylinder and inner flat tube using nanofluid. Natural convection heat transfer in two-dimensional region formed by constant heat flux horizontal flat tube concentrically located in cooled horizontal cylinder was studied numerically by using nanofluid. tube. Two-phase mixture model has been used to investigate hydrodynamic and thermal behaviors of the nanofluid over a wide range of the Grashof and Reynolds numbers. Concentration of the nanoparticles is higher at the bottom of the tube and also at the near wall region. Fully developed laminar mixed convection of a nanofluid consisting of water and Al₂O₃ in a horizontal curved tube has been studied numerically by Akbarinia and Behzadmehr [12]. Simultaneous effects of the buoyancy force, centrifugal force and nanoparticles concentration has been presented. The nanoparticles volume fraction does not have direct effects on the secondary flow, axial velocity and the skin friction coefficient. However, its effects on the entire fluid temperature could affect the hydrodynamic parameters when the order of magnitude of the buoyancy force becomes significant compared to the centrifugal force. Kalteha et al. [13] studied both numerically and experimentally the laminar convective heat transfer of an alumina-water.
nanofluid flow inside a wide rectangular microchannel heat sink. Decent investigation in the same way of the present study was performed by Matin and Pop [14]; they introduced a numerical study of mixed convection flow and heat transfer of Al2O3-water nanofluid inside an eccentric horizontal annulus with rotation on the inner cylinder.

Hamed et al.[15]. An experimental investigation has been conducted to determine the effects of the rotation, heat flux, eccentricity, and Radius ratio on the convective heat transfer of air between horizontal concentric and eccentric rotating cylinders. An experimental apparatus is designed and constructed to achieve this investigation. The measured data are presented in form of Nusselt number, Rayleigh number, eccentric ratio and Radius ratio. A numerical study of mixed convection and flow inside an eccentric horizontal annulus is presented at Mousa et al. [16]. The inner and outer cylinders rotate to generate the forced convection effect. The numerical work was carried out using ANSYS FLUENT 16. Different scenarios are explored to explain the effects of different parameters on the studied problem. These parameters are rotation ratio, eccentricity ratio, radius ratio and rotation direction. The range of the Richardson number Ri, eccentricity ratio ε, radii ratio Rr and rotation ratio ωR, are 0.1 ≤ Ri ≤ 1 (mixed convection), 0 ≤ ε ≤ 0.9, 1.5 ≤ Rr ≤ 3.5 and 0 ≤ ωR ≤ 1.5 respectively. All results were performed with constant heat flux=400 W/m2 at the inner cylinder and constant inlet velocity. The effects of eccentricity, rotation ratio, radii ratio and rotation direction on the average Nusselt number, and isotherms were investigated. Results are discussed, and are found to be in good agreement with previous works. It is also found that, the eccentricity, rotation ratio, radii ratio and rotation direction has a positive remarkable effect on the average Nusselt number. The best value of eccentricity is about 0.65 to .75. The problem and boundary conditions can be described as followed:

- Radius Ratio (R\(_r\) = \(\frac{D_i}{D_o}\)) = 2
- Length of cylinder L=0.5m (assumption for future experimental work).
- Inner cylinder has constant heat flux
- Hydraulic Diameter \(D_h = \frac{4\pi(D_o^2-D_i^2)}{4\pi(D_o+D_i)} = D_o-D_i\). 
  Using \(D_h\) as characteristic length.
- Rotating inner and outer cylinders at opposite direction
- Rotation ratio (ω\(_r\)//ω\(_o\)) ranges from 0 to 1.5.
- Inlet temperature \(T_i=300\) K.
- Eccentricity ratio (\(\varepsilon = e/b\)) = 0.5.

The aim of the present work

This paper is an extension of Mousa et al. [16] but at constant radius ratio of 2 and constant eccentricity of 0.5 at different angle of eccentricity to study the best position at which the best Nusselt number occurs.

2-2 Governing Equations

Continuity equation:

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru_r) + \frac{1}{r} \frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0
\]

Momentum equations:

R-direction

\[
\rho \left( u_r \frac{\partial u_r}{\partial r} + u_\theta \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_r^2}{r} \right) = - \frac{\partial p}{\partial r} + \rho g_r + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} \frac{\partial u_r}{\partial \theta} \right]
\]

2-1 Physical Model

The physical model, which describes the problem under investigation, is illustrated on Figure (1).
Energy equations:

\[
\begin{align*}
\rho \left( u_r \frac{\partial u_r}{\partial t} + \frac{u_r u_\theta}{r} + u_z \frac{\partial u_r}{\partial z} \right) &= -\frac{\partial p}{\partial r} + \rho g_\theta \\
&\quad + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} \right] + \frac{u_\theta}{r^2} - \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} \\
\rho \left( u_z \frac{\partial u_z}{\partial t} + \frac{u_r u_z}{r} + u_z \frac{\partial u_z}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \rho g_z \\
&\quad + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \\
\left( u_r \frac{\partial T}{\partial t} + \frac{u_r u_\theta}{r} + u_z \frac{\partial T}{\partial z} \right) &= \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right].
\end{align*}
\]

Z-direction

To insure the independence of mesh, a various number of elements is used to know the most accurate range of number of elements. Nusselt number, and skin friction coefficient are presented on Figure (2) to investigate the effect of variation of no of elements at Radius ratio=2, Eccentricity Ratio (e/b) =0.5, and Re=0.

3-Mesh independence and Validation

Figure (2) shows that the numerical results were independent of the grid when the number of cells are larger than 1,100,000. Based on this result, a 1,100,000 grid system was used in the present numerical analysis.

Numerical analysis of the heat transfer performance has been made for an eccentric annulus containing air at steady state condition. The numerical results obtained here are compared with the available data in the existing literature, as shown on Figures (3 and 4). The numerical results are first verified by comparison with the existing data. The average Nusselt number obtained from the present study is compared with the available data as indicated in Figures (3 and 4). The results of the present study show under predictions than obtained by Hamed et al.[15]. This may be due to different technique used in the solution. Hamed et al.[15] used Fluent numerical code version 6.3.26 and present study used ANSYS FLUENT 16.

Figure (3) shows the Effect of variation of Rayleigh number Ra on average Nusselt number Nu for Radius ratio=2.84, eccentricity ratio =0.16, Pr=0.7 and Re=0.
Figure (3) Effect of variation of average Nusselt number (Nu) on Rayleigh number (Ra).

Figure (4) shows the Effect of variation of Rayleigh number Ra on average Nusselt number Nu for Radius ratio=2.84, eccentricity ratio =0.46, Pr=0.7 and Re_{Ω}=0.

4-Results and discussions

The angle of eccentricity is calculated from the positive direction of x-axis as shown in Figure (5).

The effect of eccentricity angle on the average Nusselt number at constant radius ratio of 2 and constant eccentricity of 0.5 is shown in Fig. (6). The variation is almost sinusoidal and similar behavior is noticed at angle of 0° (right) an 180° (left) with maximum value occurring at angle between 180° and 270°. Calculating other two angles between 180° and 270° to determine the maximum angle at which the higher Nusselt number exists, this is shown in Figure (7).

The effect of position of eccentricity on thermal field for eccentric annulus is illustrated in Figure (8). The thermal plumes above the inner cylinder are enhanced and become tilted in the direction of rotation.
Isotherms of the two angles between 180° and 270° is illustrated in Figure (9). It is observed that as the angle increases the temperature decreases, indicating that more cooling for the cylinder surface, i.e. higher heat transfer rate with the increase of the angle between 180° and 210° and begin to decrease with increasing the angle to 270°.

Figure (8) Effect of eccentricity angle on the isotherms
\[ \theta = 180 \]
\[ \theta = 210 \]
\[ \theta = 240 \]
\[ \theta = 270 \]

\[ \omega_i = 0, \omega_o = 0 \]
\[ \omega_i = c, \omega_o = 0 \]
\[ \omega_R = 0 \]
\[ \omega_R = -0.5 \]
\[ \omega_R = -0.75 \]
\[ \omega_R = -1 \]
ωR=1.5

Figure (9) Effect of eccentricity angle on the isotherms

The calculated thermal hydraulic performance versus angle of eccentricity for different rotational ratios at opposite direction is shown in Figure (10). The figure shows that the best value of angle of eccentricity shown by the figure is about 215° and the worst value is about 65°.

Figure (10) Thermal hydraulic performance versus angle of eccentricity for different rotational ratios

5- Conclusions

The maximum value is at an angle around 215° and minimum value at an angle around 100° and what it looks like at the Fourth quarter. The average Nusselt number increases with the increase in the Rotation Reynolds number for all angles.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>b</td>
<td>Annulus gap width (r_o-r_i)</td>
</tr>
<tr>
<td>D_h</td>
<td>Hydraulic diameter</td>
</tr>
<tr>
<td>ε</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>h</td>
<td>Heat Transfer Coefficient ((w/m^2.k))</td>
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<tr>
<td>k_f</td>
<td>Thermal conductivity of fluid</td>
</tr>
<tr>
<td>L</td>
<td>Length of cylinder</td>
</tr>
<tr>
<td>q</td>
<td>Wall heat flux per unit area (w/m^2)</td>
</tr>
<tr>
<td>r_o</td>
<td>Outer radius (m)</td>
</tr>
<tr>
<td>r_i</td>
<td>Inner radius (m)</td>
</tr>
<tr>
<td>R_r</td>
<td>Radius Ratio (r_o/r_i)</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
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<tr>
<td>x,y,z</td>
<td>Cartesian coordinates</td>
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r,θ,z Cylindrical coordinates

Greek symbols

ω Angular velocity [rad/S]
ω_R Rotation ratio (ω_o/ω_i)
ε Eccentricity Ratio (e/b)
θ Angle of eccentricity
α Thermal diffusivity
μ Dynamic viscosity
β Coefficient of thermal expansion β
ν Kinematic viscosity

Dimensionless numbers

Nu Nusselt number (h*D_h/k_f)
Ri Richardson number (Gr/(Reω^2))
Pr Prandtl number

References


