Effects of Temperature Dependent Viscosity and Thermal Conductivity on MHD Mixed Convection Flow of Dusty Fluid Through a Vertical Porous Plate

Santana Hazarika¹, G.C. Hazarika²
Department of Mathematics, Dibrugarh University, Assam, India, 786004.

Abstract — This paper presents the effects of chemical reaction on MHD mixed convection heat and mass transfer of an incompressible dusty viscous fluid through a vertical porous plate with temperature dependent viscosity and thermal conductivity in the presence of suction. The fluid viscosity and thermal conductivity are assumed to be vary as inverse linear function of temperature. The governing partial differential equations are reduced to a system of ordinary differential equations by introducing similarity transformations. The non-linear similarity equations are solved numerically by applying the Runge-Kutta method of fourth order with shooting technique. The numerical results are presented graphically to illustrate influence of different values of the parameters on the velocity, temperature and concentration profiles. Skin friction, Nusselt number and Sherwood number are also computed and presented in tabular form.

Keywords— Chemical reaction, Dusty fluid, mixed convection, shooting method, vertical porous surface, variable viscosity and thermal conductivity.

I. INTRODUCTION

Dusty fluids are the fluid which combinations of fluid and fine dust particle. Such type of study is due to many applications like environmental pollution, smoke emission from vehicles, emission of effluents from industries, cooling effects of air conditioners, flying ash produced from thermal reactors and formation of raindrops etc. Saffman [12] investigated the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Soundalgekar and Gokhale [14] analyzed the flow of a dusty gas past an impulsively started infinite vertical plate with an implicit finite difference technique by using the Saffman [12] model.

MHD flow problems have become in view of its significant applications in industrial manufacturing processes such as plasma studies, petroleum industries, MHD power generator cooling of clean reactors, boundary layer control in aerodynamics. MHD flow has application in Metrology, solar physics and in motion of earth core. Also it has applications in the field of stellar and planetary magneto spheres, chemical engineering and electronics. Choudhury and Hazarika [1] studied the effects of variable viscosity and thermal conductivity on MHD flow due to point sink. Sarma and Hazarika [13] studied the effect of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence magnetic field.

Mixed convection flow is one of the transport phenomena which combination of both free and forced convection flow. The study of such type of problems are very important as the subject has wide range applications in wastewater treatment, power plant piping, combustion and petroleum transports etc. Gireesha et al. [3] studied the mixed convective flow of a dusty fluid over a vertical stretching sheet with non-uniform heat source/sink and radiation. Makinde and Sibanda [8] investigated MHD mixed- convection flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction. Ling and Dybbs [7] studied forced convection over a flat plate submerged in a porous medium with variable viscosity case.

Combined heat and mass transfer problems with chemical reaction have attracted considerable attention due to their wide range applications in many industries such as food processing and polymer production. There are two types of chemical reaction i.e homogeneous and heterogeneous reaction. A homogeneous reaction takes place uniformly in the entire given phase whereas a heterogeneous reaction exists within the boundary of a phase. Chemical reaction depends on the concentration of the species itself. Rajeswari et al. [9] investigated on chemical reaction, heat and mass transfer on non-linear MHD boundary layer flow through a vertical porous surface in the presence of suction. Krishna et al. [5] studied the magnetic field and chemical reaction effects on convective flow of dusty viscous fluid. Eldabe et al. [2] discussed the
peristaltic motion with heat and mass transfer of a dusty fluid through a horizontal porous channel under the effect of wall properties. Reddy et al. [10] analysed the effects of mass transfer and heat generation on MHD free convection flow past an inclined vertical surface in a porous medium.

The purpose of the present study is to extend the work of Rajeswari et al. [9] to the dusty fluid with variable viscosity and thermal conductivity. The flow governing equations are transformed into ordinary differential equations by using similarity transformations. The resulting equations are solved numerically by Runge-Kutta shooting method. The fluid viscosity and thermal conductivity are assumed as inverse linear functions of temperature.

II. MATHEMATICAL FORMULATION

We consider the two dimensional steady mixed convection flow of an incompressible viscous electrically conducting dusty fluid past a semi-infinite vertical porous plate with temperature dependent viscosity and thermal conductivity. \((u, v)\) and \((u_p, v_p)\) are the velocity components of fluid and dust particles along \((x, y)\) direction where \(x\)-axis is taken along the plate and \(y\)-axis normal to the \(x\)-axis. A transverse magnetic field strength \(B_0\) is imposed along the \(y\)-axis. The magnetic Reynolds number of the flow is taken to be sufficiently small so that induce magnetic field is ignored. The viscous dissipation is considered in the energy equation. The fluid properties are assumed to be constant except for the fluid viscosity and thermal conductivity.

The flow governing equations for the present problem are:

**Continuity equation for fluid phase**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

**Momentum equation for fluid phase**

\[
\frac{u u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_0} \frac{\partial}{\partial y} \left( \frac{\mu}{\rho_0} \frac{\partial u}{\partial y} \right) + K \frac{u}{m} - u
\]

\[
+ g \beta' (T - T_w) + g \beta'' (C - C_w) - \frac{\sigma B_0^2}{\rho_0} (U - u)
\]

**Energy equation for fluid phase**

\[
\frac{\partial}{\partial x} \left( \rho_0 c_p \frac{u}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( \frac{k}{\tau_T} \frac{\partial T}{\partial y} \right) + \frac{N c_p}{\tau_T} (T - T_w)
\]

\[
+ \frac{N}{\tau_T} (u - \bar{u})^2 + \mu \frac{\partial^2 u}{\partial y^2}
\]

**Concentration equation for fluid phase**

\[
\frac{\partial}{\partial x} (\rho_p u_p) + \frac{\partial}{\partial y} (\rho_p v_p) = 0
\]

**Momentum equation for dust phase**

\[
u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = \frac{K}{m} (u - u_p)
\]

\[
u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} = \frac{K}{m} (v - v_p)
\]

**Energy equation for dust phase**

\[
u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} = - \frac{c_p}{c_m \tau_T} (T_p - T)
\]

**Concentration equation for dust phase**

\[
u_p \frac{\partial C_p}{\partial x} + v_p \frac{\partial C_p}{\partial y} = D \frac{\partial^2 C_p}{\partial y^2} - k (C - C_w)
\]

where \(\mu, \rho_0, \rho_p, k\) are the coefficient of viscosity, density and thermal conductivity of fluid. \(\rho_p\) and \(N\) are the density and number density of dust particle phases. \(g, \beta'\) and \(\beta''\) are the acceleration due to gravity, thermal expansion coefficient and concentration expansion coefficient. \(k\) are the permeability of porous medium. \(C_w\) fluid temperature within the boundary layer, fluid temperature in the free stream and temperature of the dust particle. \(k_1\) and \(k_2\) are the fluid and dust chemical reaction parameters. \(C_p, C_m, k\) and \(m\) are the specific heat of fluid and dust particles, thermal conductivity and mass of the dust particle. \(C_p\) and \(C_m\) are the concentration of the fluid and dust phases. \(C_w\) is the concentration of the fluid in the free stream. \(D\) and \(D_p\) are the coefficient of mass diffusivity of the fluid and dust particle. \(\tau_T\) and \(\tau_T'\) are the thermal equilibrium time and relaxation time which the time required by a dust particle to adjust its temperature and velocity relative to the fluid. Surface temperature and concentration variation are assumed as power-law of \(x\).

The boundary conditions for the problem are

\[
\begin{align*}
 u &= U = \alpha x, \quad v = v_0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0\\
 u &= 0, \quad \frac{\partial u}{\partial y} = 0, \quad v = v_0, \quad \rho_p \rightarrow \rho_0, \quad T = T_w, \quad C = C_w \quad \text{as} \quad y \rightarrow \infty
\end{align*}
\]

where \(T_w = T_{w0} + N_1 x'\) and \(C_w = C_{w0} + N_2 x''\) are the wall temperature and wall concentration, \(N_1, N_2, n\) and \(n_1\) are positive constant. \(a\) is dimensional constant, \(\rho_0\) is the density ratio.

Lai and Kulacki [6] has assumed the fluid viscosity as

\[
\frac{1}{\mu} = \frac{1}{\mu_0} \left[ 1 + \gamma(T - T_w) \right]
\]

or,

\[
\frac{1}{\mu} = A(T - T_w)
\]
here, \( A = \frac{\gamma}{\mu_v} \), \( T_r = T_w - \frac{1}{\gamma} \)

Similarly,
\[
\frac{1}{k} = \frac{1}{k_\infty} \left[ 1 + \xi(T - T_w) \right]
\]
or,
\[
\frac{1}{k} = B(T - T_k)
\]
here, \( B = \frac{\xi}{k_\infty}, \ T_k = T_w - \frac{1}{\xi} \)

where \( \mu_v, \ k_\infty \), and \( T_w \) are the viscosity, thermal conductivity and temperature at free stream. \( A, \ B, \ T_r \), and \( T_k \) are constants and their values depend on the reference state and thermal property of the fluid.

In general \( A > 0 \) for liquids and \( A < 0 \) for gases. \( \gamma \) and \( \xi \) are constant based on thermal property of the fluid.

Let us introduce the following transformations:
\[
\begin{align*}
\phi &= \frac{C - C_w}{C_w - C_{\infty}}, \quad \phi_p(\eta) = \frac{C_p - C_w}{C_w - C_{\infty}} \\
\theta(\eta) &= \frac{T - T_w}{T_w - T_{\infty}}, \quad \theta_p(\eta) = \frac{T_p - T_w}{T_w - T_{\infty}} \\
\eta &= \sqrt{\frac{\alpha}{\nu_{\infty}}} f(\eta), \quad u_p = \alpha f(\eta), \quad v_p = \sqrt{\alpha} g(\eta), \quad \xi = H(\eta) \\

\end{align*}
\]

where
\[
T - T_w = N_1 x^n \theta \quad \text{and} \quad C - C_w = N_2 x^n \phi
\]

Using the transformations (9-15), in equations (1-7), it is seen that the equation of continuity satisfies identically and rest of the equations becomes:

\[
\begin{align*}
f''' - \frac{\theta''}{\theta} f'' + \frac{\theta'' - \theta}{\theta} f' + \frac{\beta F'}{1 - f'} + \frac{\beta \theta'}{1 - f'} = 0 \\
\theta''' - \frac{\theta''}{\theta} \frac{\theta'' - \theta}{\theta} \theta' + \frac{N \rho r_{tr}}{E_c} \left( \theta_p - \theta \right) - \frac{N \rho r_{tr}}{E_c} \left( \theta_p - \theta \right) f'' = 0
\end{align*}
\]

\[
\begin{align*}
\phi''' - S c \left( n_f \phi' f - f \phi' + \gamma \phi \right) = 0 \\
GF' + F^2 + \beta (F - f') = 0 \\
GG' + \beta (f + G) = 0 \\
HF + HG' + GH' = 0
\end{align*}
\]

\[
\begin{align*}
n_f \theta_p + G \theta_p' + \frac{c_p}{\alpha c_m \tau_v} \left( \theta_p - \theta \right) = 0 \\
\phi_p'' - S c_p \left( n_f \phi f - f \phi' + \gamma \phi \right) = 0
\end{align*}
\]

Here the dimensionless parameters are defined as:
\[
l = \frac{m N}{\rho_p} \quad \text{is the mass concentration of particle,} \quad \tau = \frac{m}{k} \quad \text{is the relaxation time of the particle phase,} \quad \beta = \frac{1}{\alpha \tau} \quad \text{is the fluid particle interaction parameter,} \quad \rho_r = \frac{\rho_p}{\rho} \quad \text{is the relative density,} \quad M^2 = \frac{\sigma B_0^2}{\rho \alpha S^2} \quad \text{is the magnetic field parameter,} \quad Gr = \frac{g \beta' U (T_w - T_{\infty}) \xi^2}{4 \nu_{\infty} v_{\infty}^2} \quad \text{is the local Grashof number,} \quad Gm = \frac{g \beta' U (C_w - C_{\infty}) \xi^2}{4 \nu_{\infty} v_{\infty}^2} \quad \text{is the mass Grashof number,} \quad Re = \frac{U x}{\nu_{\infty}} \quad \text{is the local Reynolds number,} \quad P_1 = \frac{Gr}{Re^2} \quad \text{is the temperature buoyancy parameter,} \quad P_2 = \frac{Gm}{Re^2} \quad \text{is the mass buoyancy parameter,} \quad P_e = \frac{\mu c_p}{k} \quad \text{is the Prandtl number,} \quad E_c = \frac{\alpha^2}{c_p N_1 x^{n-3}} \quad \text{is the Eckert number,} \quad S = -2v_p \left( \frac{x}{\nu_{\infty} U} \right)^2 \quad \text{is the suction parameter,} \quad Sc = \frac{\nu_{\infty}}{D} \quad \text{is the Schmidt number of fluid,} \quad Sc_p = \frac{D_{cs}}{\nu_{\infty}} \quad \text{is the Schmidt number of dust particle,} \quad \theta_r = \frac{T_r - T_{\infty}}{T_w - T_{\infty}} \quad \text{is the viscosity variation parameter and} \quad \theta_t = \frac{T_t - T_{\infty}}{T_w - T_{\infty}} \quad \text{is the thermal conductivity variation parameter.}
\end{align*}
\]

The transformed boundary conditions are:
\[
\begin{align*}
\phi = S \cdot f' = 1. \theta = 1. \phi = 1 & \quad \text{at} \quad \eta = 0 \\
\phi' = 0, \ F = 0, \ G = -f, \ H = \omega, \ \theta = 0, \ \theta_p = 0 & \quad \text{as} \quad \eta \rightarrow \infty
\end{align*}
\]
The important physical quantities are the skin friction coefficient \( C_f \), heat transfer coefficient in terms of Nusselt number \( Nu \) and Sherwood number \( Sh \) for the surface which are define by

\[
C_f = \frac{\tau_w}{\rho \nu U_w^2} = -\frac{\theta_1}{(1-\theta_\infty)} Re^{-\frac{1}{2}} f'(0)
\]

\[
Nu = \frac{q_w}{k_\infty(T_w - T_\infty)} = \frac{0}{1-\theta_\infty} Re^{-\frac{1}{2}} \theta'(0)
\]

\[
Sh = \frac{\bar{h}_m}{h_{th}(C - C_w)} = \frac{\theta_1}{1-\theta_\infty} Re^{-\frac{1}{2}} \phi'(0)
\]

where, \( \tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0} \), \( q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \),

\[
h_m = -D \left( \frac{\partial C}{\partial y} \right)_{y=0}
\]

### III. RESULTS AND DISCUSSION

Runge-Kutta fourth order method along with shooting technique has been used to solve the boundary value problems (14) - (22) numerically. The following numerical values of various parameters are taken as \( Pr = 0.7, \theta_1 = 2, \theta_\infty = 2, \beta = 0.5, M = 0.5, A = 0.2, P_1 = 0.1, P_2 = 0.1, Ec = 0.05, Sc = 0.6 \). Velocity, temperature and concentration profiles of both fluid and dust particles for different parameters are shown graphically.

In figure 1 to figure 4, the velocity profiles of both fluid and dust particles are plotted against \( \eta \) for various combinations of parameters. Figure 1 shows the velocity profile of both the fluid and dust phases for various values of fluid particle interaction parameter \( \beta \). The fluid velocity decreases but fluid velocity increases with increasing values of fluid particle interaction parameter.

In figure 2, the velocity profiles of both fluid and dust phases are plotted for various values of velocity variation parameter \( \theta_1 \). It is observed that the velocity distributions of both fluid and dust phases decrease with increases of velocity variation parameter \( \theta_1 \). Physically it means that the viscous force due to viscosity opposes the relative motion between their layers.

Figure 3 illustrates the distribution of both fluid and dust phases velocity for various particle mass concentration parameter \( \eta \). The velocity of both phases decreases with increasing values of \( \eta \). It is because that these dust particles oppose the motion of the fluid.

Unless otherwise stated, we have taken \( \tau_\infty = \tau_v = 0.5 \), \( c_p = c_m = 0.2, \rho = 0.5 \) and \( \alpha = 1 \) for numerical calculation. In figure 4, the temperature profiles of both fluid and dust phases are displayed for various values of viscosity variation parameter \( \theta_1 \). From this figure we can observed that the temperature of both fluid and dust phases increase with increases of viscosity variation parameter \( \theta_1 \). That means, increase in viscosity enhances the friction between the layers so temperature increases.

Figures 5 are plotted for the temperature profiles of both the fluid and dust phases for different values of fluid particle interaction parameter \( \beta \). It is clearly observed from this figure that the temperature of both fluid and dust phases decrease with the increasing values of fluid particle interaction parameter \( \beta \).

It is clearly observed from figure 6 that the temperature of both fluid and dust phases decrease with the increases of thermal conductivity parameter \( \theta_\infty \). Physically it means that increase in thermal conduction enhances the transportation of heat from a hot region to an adjacent colder region. Since temperature within the boundary layer is more than the outside so temperature becomes less.

The effects of the viscous dissipation parameter i.e. the Eckert number \( Ec \) on temperature are shown in fig 7. It is observed that the temperature increases with increases of Eckert number \( Ec \). Physically, this is due to fact that the heat energy is stored in the considered fluid due to frictional heating.

The effect of the Schmidt number on concentration profiles are shown in figure 8. Schmidt number represents the ratio of momentum diffusivity to molecular diffusivity. Physically it relates the hydrodynamic boundary layer and mass transfer boundary layer. It is concluded that the increase in Schmidt number decreases the concentration. From figure 9, it is that the concentration decreases with increase of chemical reaction parameter.

In table 1, we have compared the present numerical results of the missing values \( f^\star(0), \phi'(0) \) and \( \theta'(0) \) with those of values obtained by Rajeswari et al. [4] for various values of Prandtl number \( Pr \). From this table we have observed that the accuracy of \( f^\star(0), \phi'(0) \) and \( \theta'(0) \) of present study are very significant than that of Rajeswari et al [4].

In table 2 and table 3, the missing values \( f^\star(0), \phi'(0) \) and the coefficient of skin friction \( C_f \), Nusselt number \( Nu \) and Sherwood number \( Sh \) are evaluated for different values of \( \theta_1, \theta_\infty, \beta, M \). It is clear from these tables that the skin friction decreases with the increase of \( \theta_1, \theta_\infty, M \) but increases with the increase of \( \beta \). Nusselt number increases with the increase of \( \theta_\infty \) but decreases with the increase of \( \theta_1, M, \beta \). Sherwood number \( Sh \) decreases for the increasing values of \( \theta_\infty \).
Figure 1: Velocity profile for different $\beta$.

Figure 2: Velocity profile for different $\theta_r$.

Figure 3: Velocity profile different $l$.

Figure 4: Temperature profile for different $\theta_r$.

Figure 5: Temperature profile for different $\beta$.

Figure 6: Temperature profile for different $\theta_k$.

Figure 7: Temperature profiles for different $Ec$.

Figure 8: Concentration profile of dust phase for different $Sc_p$. 
IV. CONCLUSIONS

From the above analysis, it is clear that the viscosity and thermal conductivity variation parameter along with the other parameters such as fluid particle interaction parameter $\beta$, magnetic parameter $M$ and Schmidt number etc. have significant effects on velocity, temperature and concentration within the boundary layer. We can conclude from this analysis as:

1. The increasing values of viscosity retard the velocity for both fluid and dust phase.
2. Temperature of fluid phase is higher than that of the dust phase as well as parallel.
3. Increasing the value of Eckert number enhances the temperature of both fluid and dust phases.
4. The velocity of fluid and dust particles decreases with increase mass concentration of particle.
5. Concentration of both phases decreases with increase chemical reaction parameters.
6. Both skin friction and Nusselt number decrease with increase of viscosity.

Table 1:

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0.6  -0.01137  -0.01008  -0.0046  -0.07194  0.006378  0.148143

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