Modified Decomposition Method for Solution Partial Differential Equations with Derivative Boundary Conditions

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Abstract *- In this paper, modified decomposition method of solution the partial differential equation two-sided has been presented. Simulation results for example illustrate the comparison of the analytical and numerical solution. The results were presented in tables using the MathCAD 12 software package when it is needed.*

Keywords: *Modified decomposition method, derivative boundary condition problem, Partial differential equation.*

I. INTRODUCTION

Since its introduction by G. Adomian, the adomian decomposition algorithm has been used in finding numerical solutions to a wide variety of problems in mathematics, physics and engineering. It is fundamentally based on providing a solution in the form of series and decomposing nonlinear terms using adomian polynomials [1-2]. In recent years adomian's algorithm has been modified to make it more effective in providing solutions to differential and integral equations. Yahya Qaid Hassan and Liu Ming Zhu [3]-[4] used a modified adomian Decomposition method in solving singular boundary value problems of higher ordinary differential equations. S. N. Venkatarangan and T. R. Sivakumar [5] adopted a modified decomposition method for boundary value problems. A. M. Wazwaz [6] combined the adomian decomposition method and Pade Approximants in solving Flierl-petviashivili equation and its variants. In 2001, Khuri [7] proposed the Laplace Decomposition method which was later developed by Yusufoglu [8] in 2006. A reliable modification of this decomposition method was achieved by Khan [9] in 2009. Our current work is motivated by Yasir Khan and Naeem Farz [10] and Rashidi, M. M. [11] who applied the modified Laplace decomposition method in solving boundary layer equation. In this work we applied the Laplace.
Decomposition method to obtain a series solution of Blasius' boundary layer equation for the flat plate. It is interesting to note here that the series solution obtained by our method is exactly the same as that obtained by Weyl 1942a, [12].

In this paper, we present modified decomposition method for solving the partial differential equation two-sided with derivative boundary conditions:

$$\frac{\partial}{\partial t}\mu(x,t) = \frac{\partial^2}{\partial_{+}x^2}\mu(x,t) + \frac{\partial^2}{\partial_{-}x^2}\mu(x,t) - \mu(x,t) + h(x,t) \qquad (1)$$

with the initial condition

$$\mu(x,0) = f(x,y), 0 \le x \le T$$

and the derivative boundary conditions

$$\mu_x(0,t) = v_1(t), 0 < t \le T$$
$$\mu_x(1,t) = v_2(t), 0 < t \le T$$

Where f, v_1, v_2, μ and h are known functions, T is given constant. In the present work, we apply the modified Adomian's decomposition method for solving eq.(1) and compare the results with exact solution.

II. PROPOSED METHOD

In this paper, we use modified decomposition method for solving partial differential equations two-sided with derivative boundary condition given in eq. (1). In this method we assume that:

$$\mu(x,t) = \sum_{n=0}^{\infty} \mu_n(x,t)$$

eq.(1) can be rewritten:

$$L_t\mu(x,t) = L_{+xx}\mu(x,t) + L_{-xx}\mu(x,t) - \mu(x,t) + k(x,t)$$
(2)

Where

$$L_t(\cdot) = \frac{\partial}{\partial t}(\cdot) \text{ and } L_{xx} = \frac{\partial^2}{\partial x^2}$$

The inverse $\quad L^{-1} = \int_0^t (\cdot)dt \qquad (3)$

$$L^{-1}(L_t\mu((x,t))) = L^{-1}(L_{+xx}(\mu(x,t)) + L_{-xx}(\mu(x,t))) - L^{-1}(\mu(x,t)) + L^{-1}(k(x,t))$$

Then, we can write,

$$\mu(x,t) = \mu(x,0) + L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\infty}\mu_n\right) + L_{-xx}\left(\sum_{n=0}^{\infty}\mu_n\right)\right) - L_t^{-1}(\mu(x,t)) + L_t^{-1}(k(x,t)) \qquad (4)$$

The modified decomposition method was introduced by Wazwaz [13]. This method assumes that the function $\delta(x)$ can be divided into two parts, namely $\delta_1(x)$ and $\delta_2(x)$. Under this assumption we set

$$\delta(x) = \delta_1(x) + \delta_2(x)$$

Then the modification

$$\mu_0 = \delta_1$$
$$\mu_1 = \delta_2 + L_t^{-1}(L_{+xx}\mu_0) + L_t^{-1}(L_{-xx}\mu_0) - L_t^{-1}(\mu_0)$$

$$\mu_{n+1} = L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\infty}\mu_n\right) + L_{-xx}\left(\sum_{n=0}^{\infty}\mu_n\right)\right) - L_t^{-1}\left(\sum_{n=0}^{\infty}\mu_n\right)$$
, n>1

EXAMPLE 1: Consider the following:

$$\frac{\partial}{\partial t}\mu(x,t) = \frac{\partial^2}{\partial_+ x^2}\mu(x,t) + \frac{\partial^2}{\partial_- x^2}\mu(x,t) - \mu(x,t) + 2t + t^2 + x$$

subject to the initial condition

$\mu(x,0) = x^2 \quad x \in (0,1), \quad 0 \le t \le T$

and the derivative boundary conditions

$\mu_x(0,t) = t^2, 0 < t \le T$

$\mu_x(1,t) = 1 + t^2, 0 < t \le T$

we obtain after apply the above proposed method on:

$\mu_0(x,t) = x^2 + t^2$

$\mu_1(x,t) = 0$

$\mu_2(x,t) = 0$

$\mu_3(x,t) = 0$

Then the series form is given by:

$$\mu(x,t) = \mu_0(x,t) + \mu_1(x,t) + \mu_2(x,t) + \mu_3(x,t) = x^2 + t^2$$

This is the exact solution: $\mu(x,t) = x^2 + t^2$

Table 1 shows the proposed method of solutions partial differential equation with derivative boundary condition obtained for different values.

Table1. Comparison between exact solution and proposed method for solution example (1)

x	t	Exact Solution	proposed Method	$\lvert \mu_{ex} - \mu_{MADM} \rvert$
0	5	250	250	0.0000
0.1	5	25.01	25.01	0.0000
0.2	5	25.04	25.04	0.0000
0.3	5	25.09	25.09	0.0000
0.4	5	25.16	25.16	0.0000
0.5	5	25.25	25.25	0.0000
0.6	5	25.36	25.36	0.0000
0.7	5	25.49	25.49	0.0000
0.8	5	25.64	25.64	0.0000
0.9	5	25.81	25.81	0.0000
1	5	26.00	26.00	0.0000

III. CONCLUSION

In this paper, we have applied the modified decomposition method for the solution partial differential equation two-sided heat equation with derivative boundary condition. This algorithm is simple and easy to implement. The obtained results confirmed a good accuracy of the method.

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