

Effect of Optimization Tool Approach on Linear Programming Methods to Optimize Mathematical Manipulation

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Abstract: The main target of this work is based on the effect of optimization tools on linear programming methods to optimize the mathematical calculations. Linear programming plays an important role in our lives. There are various methods to solve LPP, such as simplex, dual-simplex, Big-M, two phase and graphical method. In this, an approach is presented to solve LPP by considering the optimization tool of MATLAB and compare it with tabular methods of LPP. The complexity reduction is done by eliminating the large number of steps. By using proposed technique, the calculation part has been completely avoided and we can achieve the results in considerable duration. The objective function of linear programming problem (LPP) involves in the maximization and minimization problem with the set of linear equalities and inequalities constraints. By using optimization tool in MATLAB used for LPP, reduced to form of Linear programming (LP) problem. So practically, for large number of constraints & variables, it is not possible to solve these problems by tabular method.. It takes more computation time & iterations.. By using proposed technique, we can achieve the results in considerable duration & exact optimum solution.

Keywords: Linear programming problem, optimization tools, optimal solution, Tabular Method.

I. INTRODUCTION

Optimization technique is a mathematical approach to solve the problem for finding the best possible solution out of the available alternatives under the given circumstances. There are various applications of optimization in engineering field like in electrical

engineering, civil engineering, mechanical engineering etc as well as non engineering. applications like business, agriculture, manufacturing, production, transportation, investment policy etc. An application of optimization

technique in all the fields to determine the best possible solutions to minimize the total present cost that will maximize the total profit and to minimize the total losses.

Linear programming is the most prominent optimization technique is applicable for the solution of real life problems in which the objective function and constraints are the linear functions of the decision variables. If there is no squared term, trigonometric functions, ratios of variables, then the problem is called a Linear Programming (LP) problem. It can be defined as a mathematical technique to determine the best optimal solution of any industrial, research problem to achieve best outcomes. The resources may be man, material, machine, land, etc. Most real life problems when formulated as an LP model consisting of more than two variables and therefore there is need of more efficient method to get optimal solution. So Optimization tool in MATLAB for LPP is very efficient, time consuming, most accurate tool to solve

the LPP. For tabular calculation, it is necessary to identify separately and to solve by using according to the algorithm.

II. Algorithms Used

A. Simplex method

Simplex method is the most accurate method to solve the Linear Programming problem. It can be modified according to the nature of problem. It can be easily implemented on computers. It overcomes the limitations of graphical method. It can be solved with lesser number of constraints and work with less than or greater than or equal to constraints. It uses a tabular form of representing the numbers.

Algorithm:

1. The problem should be of maximization type, if it is minimization then convert it into maximization by multiplying -1 to the objective function.

2. All the constraints should be of less than or equal to type, if it is greater than then convert it into less than by multiplying -1 to the constraints.
3. The RHS of the constraints should be positive/zero
4. All the less than or equal to constraints get converted into equality constraints by adding non negative variable known as slack variable.
5. After completing the 1st simplex table, check that if all the elements in the row are $c_j - \sum e_i \cdot a_{ij}$ are negative, then optimum solution is reached.
6. If some elements in the row $c_j - \sum e_i \cdot a_{ij}$ are positive/negative/zero, then further optimization is possible.
7. Select the largest positive value in the row $c_j - \sum e_i \cdot a_{ij}$.
8. Calculate the value of Θ
9. Select the smallest positive value in the column of Θ
10. Select the pivot element
11. Form the next simplex table upto getting all the values in the row $c_j - \sum e_i \cdot a_{ij}$ are negative.

B. Two phase simplex Method

This is the case when the linear programming problem does not have slack variables for some of the equations or when the slack variables have negative coefficients.

Algorithm:

1. The problem should be of maximization type.
2. The right hand side of the constraints should be positive or zero.
3. All the constraints should be of less than or equal to or greater than or equal to type.
4. The less than or equal to type constraints are converted into equality constraints by adding non negative variable called as slack variable.
5. The greater than or equal to type constraints are directly converted into equality constraints by subtracting non negative variable to the LHS called as surplus variable and added one more variable called as artificial variable.

Phase –I:-

1. Here objective function is taken as addition of all the artificial variables.

$$\text{Min } w = A_1 + A_2 + \dots$$
2. The starting basics for phase –I are slack and artificial variables.
3. For the first simplex table, Select the largest negative value in the row $c_j - \sum e_i \cdot a_{ij}$
4. Calculate the value of Θ
5. Select the smallest positive value in the column of Θ
6. Select the pivot element
7. After finding optimum solution to auxiliary objective function if $\text{min } w > 0$, then there is no feasible solution to the problem. the method or solution is terminated.
8. If all the values in the row $c_j - \sum e_i \cdot a_{ij}$ are positive and $\text{min } w = 0$, then proceed for phase II

Phase –II:

1. In this case original objective function is considered.
2. The first simplex table for phase II is formed as, The coefficient of a_{ij} & b are picked up from last table of phase I
The column corresponding to the artificial variable is dropped
The entries in the row $c_j - \sum e_i \cdot a_{ij}$ are made using original objective function
3. The phase two is exactly same as a simplex method.
4. If all the values in the row are $c_j - \sum e_i \cdot a_{ij}$ are negative, then optimum solution is reached.

C. Graphical Method

A linear programming problem with only two variables presents a simple case for which the solution can be derived using a graphical method, provides a pictorial representation of the solution process.

Algorithm:-

1. Represent the given problem in mathematical form i.e. formulate the mathematical model for the given problem.
2. Plot each of the constraints on the graph. The constraints whether equations or inequalities are plotted as equation. For each constraints, assign any arbitrary value to one variable and get the value of other variable and vice versa.. Plot these two points and connect them by a straight line. Thus each constraint is plotted as line in first quadrant.
3. Identify the feasible region (or solution space) that satisfies all the constraints simultaneously.
4. For \geq type constraints, the area on or above the constraints line i.e. away from the origin and for \leq type constraints the area on or below the constraints line i.e. towards the origin. The area common to all the constraints is called as feasible region and is shown shaded. Any point on or within the shaded region represent the feasible region to the given problem.
5. Find the coordinates of each extreme point of the feasible region. Find the value of the objective function at each extreme point. The point at which objective function is maximum or minimum is the optimal point and its coordinates gives optimal solution

D. Dual simplex Method:

Simplex method maintains the feasibility during successive iterations whereas dual simplex method maintain the optimality.

Algorithm:

1. The problem should be of maximization type
2. Right hand side of the constraints should be positive, negative or zero.
3. All the constraints should be of less than or equal to type.
4. Convert all the inequality constraints into equality constraints by adding a slack variable.

5. Select the largest negative value in the column of b
6. Calculate the value of Θ ($c_j - \sum e_i \cdot a_{ij}$ divided by values in the row of largest negative number).
7. Select the smallest positive value in the row of Θ
8. Select the pivot element
9. If any element in the row $c_j - \sum e_i \cdot a_{ij}$ are positive, then the method fails.
10. If all the values in the row $c_j - \sum e_i \cdot a_{ij}$ are negative and all the values of column of b are positive, then optimum solution is obtained.
11. If all the values in the row of Θ are negative, then solution space is unbounded.

III. Implementation of Tabular Methods

Simplex Method

Case Study:

Consider a chocolate manufacturing company which produces three types of chocolates A, B & C. All the chocolates require milk, choco and cashunut only. To manufacture each unit of A, B and C following quantities is required. Each unit of A requires 3 unit of milk, 6unit of choco and 3 units of cashunut. Each unit of B requires 1unit of milk and 2 units of choco and 3 units of cashunut. Each unit of C requires 3 unit of milk, 2 units of choco and no units of cashunut. The company kitchen has a total of 22 units of milk, 14 units of choco and 14 units of cashunut. On each sale the company makes a profit of Rs. 1 per unit A sold, Rs. 4 per unit B sold and 5 per unit of C sold. Now company wishes to maximize its profit. How many units of A, B and C should it produced respectively.

Solution: Let x_1 , x_2 & x_3 be the number of units produced by chocolate A B & C respily.

Types of chocolate	Milk	Chocó	Cashunut	Profit per unit
A	3	6	3	1
B	1	2	3	4
C	3	2	0	5
Limits (Unit)	22	14	14	

Maximize $f = x_1 + 4x_2 + 5x_3$

STC

$3x_1 + 6x_2 + 3x_3 \leq 22$

$x_1 + 2x_2 + 3x_3 \leq 14$

$3x_1 + 2x_2 \leq 14$

$x_1, x_2, x_3 \geq 0$

Standard Form :

Maximize $f = x_1 + 4x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

STC

$3x_1 + 6x_2 + 3x_3 + s_1 = 22$

$x_1 + 2x_2 + 3x_3 + s_2 = 14$

$3x_1 + 2x_2 + s_3 = 14$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

1st iteration:

E _i	C _j	1	4	5	0	0	0		
	Basic	X ₁	X ₂	X ₃	S ₁	S ₂	S ₃	B	Θ
0	S1	3	6	3	1	0	0	22	7.33
0	S2	1	2	3	0	1	0	14	4.66
0	S3	3	2	0	0	0	1	14	
$\sum e_i \cdot a_{ij}$		0	0	0	0	0	0		
$c_j - e_i \cdot a_{ij}$		1	4	5	0	0	0		

2nd iteration:

E _i	C _j	1	4	5	0	0	0		
	Basic	X ₁	X ₂	X ₃	S ₁	S ₂	S ₃	b	Θ
0	S1	2.01	4.02	0	1	-0.99	0	8.02	1.99
0	S2	0.33	0.66	1	0	0.33	0	4.66	7.06
0	S3	3	2	0	0	0	1	14	7
$\sum e_i \cdot a_{ij}$		1.65	3.3	5	0	1.65	0		
$c_j - e_i \cdot a_{ij}$		-0.65	0.7	0	0	-1.65	0		

3rd iteration:

e _i	C _j	1	4	5	0	0	0		
	basic	X ₁	X ₂	X ₃	S ₁	S ₂	S ₃	b	Θ
0	S1	0.5	1	0	0.24	-0.24	0	1.99	
0	S2	0	0	1	0.15	0.48	0	3.34	
0	S3	1	0	0	-0.48	0.48	1	10.02	
$\sum e_i \cdot a_{ij}$		2	4	5	1.71	1.44	0		
$c_j - e_i \cdot a_{ij}$		-1	0	0	-1.71	1.44			

As all the elements in the row $c_j - e_i \cdot a_{ij}$ are negative, hence optimum solution is obtained.

$x_1 = 0, x_2 = 1.99, x_3 = 3.34$

Maxf = 24.66

Two Phase Simplex Method

Maximize $f = x_1 + 3x_2$

STC

$-4x_1 + 3x_2 \leq 12$

$x_1 + x_2 \leq 7$

$x_1 - 4x_2 \geq 2$

$x_1, x_2 \geq 0$

Standard Form :

Maximize $f = x_1 + 3x_2$

STC

$-4x_1 + 3x_2 + s_1 = 12$

$x_1 + x_2 + s_2 = 7$

$x_1 - 4x_2 - s_3 + A = 2$

$x_1, x_2, s_1, s_2, s_3, A \geq 0$

Phase I: Objective function = Min w = A

By using the mentioned algorithm, we can find the solution .

As all the elements in the row $c_j - \epsilon_i * a_{ij}$ are positive , hence optimum solution to the auxiliary function is reached.

$$\text{Min}w = A = 0$$

So Proceed for phase II

Phase II:

Objective function : $x_1 + 3x_2$

As all the elements in the row $c_j - \epsilon_i * a_{ij}$ are negative , hence optimum solution t is reached.

$$\mathbf{X1 = 6 , x2 = 1}$$

$$\mathbf{Maxf = x1 + 3x2 = 6+3*1 = 9}$$

C. Dual simplex Method

Minimize $f = 10x_1 + 6x_2 + 2x_3$

STC

$$-x_1 + x_2 + x_3 \geq 1$$

$$3x_1 + x_2 - x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0$$

Standard Form :

Maximize $f = - 10x_1 - 6x_2 - 2x_3 + 0S_1 + 0S_2$

STC

$$x_1 - x_2 - x_3 + s_1 = - 1$$

$$-3x_1 - x_2 + x_3 + s_2 = - 2$$

$$x_1, x_2, x_3 \geq 0$$

After solving the above problem by using mentioned algorithm , in 3rd iteration ,all the elements in the row $c_j - \epsilon_i * a_{ij}$ are negative & all the values in the column of b are positive , hence optimum solution is obtained.

$$\mathbf{X1 = 0.25 , x2 = 1.25,}$$

$$\mathbf{Min f = 10}$$

D. Graphical Method

Problem:

Maximize $f = 2x_1 + 5x_2$

STC

$$8x_1 + 4x_2 \leq 15$$

$$3x_1 + 5x_2 \leq 12$$

$$x_1 \leq 5$$

$$x_1, x_2 \geq 0$$

Solution:

First find two points on the line

$$1. \quad 8x_1 + 4x_2 \leq 15 , (1.7, 3.7)$$

$$2. \quad 3x_1 + 5x_2 \leq 12 , (4, 2.4)$$

$$3. \quad x_1 \leq 5 , (5, 0)$$

By plotting graph using above coordinates, optimum solution is obtained.

$$\mathbf{A = (0, 0), B = (0, 2.4), C = (0.9, 1.8), D = (1.7, 0)}$$

$$\mathbf{ZA = 0 , ZB = 12 , ZC = 10.8 , ZD = 3.4}$$

$$\mathbf{Max Z = 12 \text{ at } x_1 = 0 \ \& \ x_2 = 2.4}$$

IV. Computational Difficulties

Any Linear Programming problem can be solved by using the Simplex method, two phase simplex method , dual simplex method , revised simplex method , Big M method , graphical method etc.

When we apply all these tabular methods, it requires too much computation time to solve large-scale problems. There is possibility to get inaccurate solution because of complicated calculations. If there are large number of variables and constraints, then calculation part makes complicated. So it is necessary to apply another method, which reduces computation time & iterations and also get the exact optimum solution.

In MATLAB, there are various optimization tools are available, that can apply to linear programming, nonlinear programming problem of optimization to get the optimum solution in less time and iterations.

V. Proposed Methodology

Linear programming consists of various methods of optimization technique like simplex, two phase simplex, dual simplex and graphical method etc. There are separate algorithms to solve each method by using tabular calculations.. If there are more number of variables in the problem, then it is not possible to solve these problem by using tabular method. It takes more iteration & more computation time & possibility to get inaccurate solution. If there

are such types of problems, then it is necessary to implement another simplified method to get the accurate & optimal solution, which takes less iteration & less computation time.

In MATLAB, there are various tools are available to solve the each & every complicated problem. To solve the linear programming problem, optimization tools are available in MATLAB. The advantage of these optimization tools is that, it takes very less computation time & iteration. & there is no chance to get the inaccurate solution. Another advantage is that , When we solve the linear programming problem by using tabular method or calculations , first there is need to identify the separate methods of LPP like simplex , two phase simplex , dual simplex, graphical etc according to the algorithms. But when we use the optimization tools in MATLAB , there is no need to identify each method separately only to identify it is linear or non linear programming problem & according to that apply the proper syntax for coding.

VI. Implementation of Proposed Methodology

To solve the any linear programming problem , the optimization toolbox is available in MATLAB. There is specific syntax for the linear programming problem is as follows

Syntax for LPP

$x = \text{linprog}(f,A,b)$
 $x = \text{linprog}(f,A,b,Aeq,beq)$
 $x = \text{linprog}(f,A,b,Aeq,beq,lb,ub)$

Table I: LPP Parameters

Sr. No.	Parameter	Description
1	F	Objective function (Coefficient vector)
2	A	Linear inequality constraints (LHS of the constraints)
3	B	Linear inequality constraints (RHS of the constraints)
4	Aeq	Linear equality constraints(LHS of the constraints)
5	Beq	Linear equality constraints (RHS of the constraints)
6	Lb	Lower bound
7	Ub	Upper bound

To solve the any linear programming problem by using optimization tool in MATLAB, it is necessary to convert given problem into standard form means if it is maximization then convert the it into minimization type . If constraints are of greater than equal to type then convert it into less than or equal to type

VII. Analysis of LPP Methods using optimization Toolbox

Table II: Optimization Toolbox

	Simplex Method
Given Problem	Max $f = x_1 + 4x_2 + 5x_3$ STC $3x_1 + 6x_2 + 3x_3 \leq 22$ $x_1 + 2x_2 + 3x_3 \leq 14$ $3x_1 + 2x_2 \leq 14$ $x_1, x_2, x_3 \geq 0$
Step :1 Standard form	Min $f = -x_1 - 4x_2 - 5x_3$ STC $3x_1 + 6x_2 + 3x_3 \leq 22$ $x_1 + 2x_2 + 3x_3 \leq 14$ $3x_1 + 2x_2 \leq 14$ $x_1, x_2, x_3 \geq 0$
Step2: MATLAB coding	Input data $f=[-1 \ -4 \ -5];$ $A=[3 \ 6 \ 3; 1 \ 2 \ 3; 3 \ 1 \ 0];$ $b=[22 \ 14 \ 14];$ $lb=[0 \ 0 \ 0];$

	$[x,fval]=\text{linprog}(f, A, b, [], [], lb)$
Step3 Result	$X= 0, 2, 3.33$ $fval=-24.66$

In similar way , we can find the optimum solution for two phase simplex , dual simplex , big M , graphical method etc. by using optimization tools in MATLAB used for LPP.

Analysis of LPP methods like simplex , two phase simplex, dual simplex and graphical method using tabular method takes more computation time & iterations. And also possibility to get inaccurate solution because of more calculations. But when Analysis of LPP methods like simplex , two phase simplex, dual simplex and graphical method using optimization tools in MATLAB takes less computation time & get accurate solution and there is no possibility to get the inaccurate solution because of calculation part is completely avoided .

Also using optimization tools for analysis of LPP methods like simplex , two phase simplex, dual simplex and graphical method , we can use same syntax for all the methods of LPP.

There is no need to apply different syntax for each methods of LPP . But in manual calculation, it is not possible to use same algorithm , there are different algorithms for each methods . So before applying LPP technique first we have to identify the methods of LPP separately and after that according to the algorithm, use the method.

So from the above discussion it is clear that, proposed methodology is the best & exact optimum method to find the solution of linear programming Problem than the tabular methods.

VIII. Conclusion:

From the above discussion, it is clear that, For linear programming problem consisting of large number of design variables and constraints, tabular methods are more complicated & takes more computation time & iterations and possibility to get inaccurate solution because of more calculations . It is important to show the effects of proposed methods on linear programming problem to get the exact optimum solution in minimum time & iterations. Proposed methodology i. e optimization tools in MATLAB used for LPP is the best , more useful , less computation time & most accurate method. Many real world problems like industrial , transportation , production , Research etc consisting of large number of variable & constraints can be easily solve by using optimization tools in MATLAB Therefore we can say that , proposed methodology i. e optimization tools in MATLAB is the

best method to solve any linear programming problem. We have also explored the use of LPP as an economic tool for sensitivity analysis to reduce the complexity over computation using introduced algorithm.

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