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Original Article

Probabilistic Analysis of Power Transformers in a Power Distribution Company with Six Types of Failures and Inspection

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Abstract - *This paper presents a probabilistic analysis of power transformers in a power distribution company. Five years of failure-maintenance data have been collected from the operations and maintenance department of the company. Six failure modes are noted after inspection, viz., bushing failure, on-load tap changer failure, cooling system failure, winding failure, tank failure, and unidentified failure. The system is analysed using Markovian and regenerative processes, and relevant reliability indices are obtained. Sensitivity analysis is performed to assess the effect of various parameters on the reliability indicators. Relevant statistical inference and life data analysis are also presented.*

Keywords - *Reliability, Availability, Inspection, Markov processes, Regenerative processes.*

1. Introduction

Power transformers are critical components of any power distribution network. Any type of failure in the power transformers causes interruption to the power supply. Being highly sophisticated and complex in nature, their failure results in high maintenance costs. For this reason, studying the performance of power transformers with a reliability perspective is essential to maximize the availability and minimize the maintenance cost.

Many efforts were put in place by several authors to study the performance of various industrial systems under different operating conditions and assumptions. Rizwan et al. (2008) studied the reliability of a single-unit programmable logic controller with inspection. Dhankar et al. (2012) presented reliability modelling and profit analysis of a system with different failure modes and replaceable servers subject to inspection.

Mathew et al. (2009; 2010; 2011 a, b) worked on a continuous casting plant with different loading capacities of cranes. An extensive probabilistic analysis was carried out by Padmavathi et al. (2012; 2013 a, b; 2014 a, b; 2015) for an evaporator of desalination with different operating conditions and assumptions. Taj et al. (2017 a, b, c; 2018 a, b, c, d, e; 2020) extensively studied a cable manufacturing plant in Oman and calculated various reliability measures along with the cost-benefit analysis of the system. Al Rahbi et al. (2017 a, b; 2018 a, b; 2019 a, b, 2020) discussed in detail various reliability models of an anode rodding plant in the aluminium industry, portraying different operating scenarios of the plant.

The previous review paper 2021 of this work summarizing the reliability modelling and analysis techniques of various complex industrial systems. Rizwan and Taj (2021) performed reliability modelling and analysis of a port programmable logic controller with five types of failure. Rizwan et al. (2022) presented a reliability analysis of three pumping units, considering all the pumps together as a single system. Sensitivity analysis was proposed by Sachdeva et al. (2022) while studying an insured system with extended conditional warranty. Rizwan et al. (2023) presented the reliability and sensitivity analysis of a membrane biofilm fuel cell.

Another past study 2023 of this work presented the reliability analysis of power transformers, considering all transformers together as one single system with four generalised failure categories (minor, major, partial, complete) and a single repairman. However, to identify the specific type of failure, viz. bushing failure, cooling system failure, winding failure, OLTC failure, tank failure, and unidentified failure, it is more reasonable to consider the inspection time spent for achieving better-optimised reliability indices and categorising the data under component failures rather than generalized failures. [1-11]

Table 1. Number of faults

S. No	Fault type	No. of Faults
1	Bushing failure	8
2	Cooling system failure	5
3	Winding failure	2
4	Tank failure	3
5	OLTC failure	5
6	Unidentified failure	6

Thus, in this paper, a probabilistic analysis of the power transformers in the Dhofar power company of Oman is discussed with inspection time and six types of failures to investigate the performance of the system. The system is analysed using Markovian and regenerative processes. Important reliability indices are obtained to assess the system's effectiveness. Sensitivity analysis is also performed to observe the influence of the parameters on the reliability indices (Tables 5, 6 and 7). Relevant statistical inference has been included and life data analysis is also presented (Tables 8 and 9). Section 6 concludes the results obtained and suggests some recommendations.

2. Materials and Methods

2.1. Data Summary

Real downtime data of the power transformers are collected from the Dhofar power distribution company of Oman. Six types of failures are noted and have been categorized accordingly, viz., bushing failure, cooling system failure, winding failure, OLTC failure, tank failure and unidentified failure. Table 1 summarizes the number of faults that occurred according to failure types between January 2017 and February 2021.

2.2. Model Description and Assumptions

The following states describe the system:

S_0: System is operative.

S_1: System is under inspection.

S_2: Failed state due to bushing failure.

S_3: Failed state due to cooling system failure.

S_4: Failed state due to winding failure.

S_5: Failed state due to OLTC failure.

S_6: Failed state due to tank failure.

S_7: Failed state due to unidentified failure.

The following operating conditions and assumptions are considered: l transformers taken together are considered as a single system.

- All transformers taken together are considered as a single system
- System is in operative mode at state 0.
- Inspection is carried out as soon as the system enters the failed state 1.
- Depending on the type of failure, the system transits from state 1 to any one of the states from 2 to 7 with failure probabilities p_1 to p_6 respectively.
- Failure times are exponentially distributed, while the repair times are assumed to be arbitrarily distributed.
- After each repair/replacement, the system works as good as new.

The state transition table of the system is shown in Table 2. The following are noted from Table 2:

- The states S_i $(i = 1, 2, 3, 4, 5, 6, 7)$ are failed states.
- The states S_i $(i = 0, 1, 2, 3, 4, 5, 6, 7)$ are regenerative states.
- 0 denotes no transition to the mentioned state.

Table 2. State transition table

S_jS_i	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_0	0	λ	0	0	0	0	0	0
S_1	0	0	$p_1 i(t)$	$p_2 i(t)$	$p_3 i(t)$	$p_4 i(t)$	$p_5 i(t)$	$p_6 i(t)$
S_2	$g_1(t)$	0	0	0	0	0	0	0
S_3	$g_2(t)$	0	0	0	0	0	0	0
S_4	$g_3(t)$	0	0	0	0	0	0	0
S_5	$g_4(t)$	0	0	0	0	0	0	0
S_6	$g_5(t)$	0	0	0	0	0	0	0
S_7	$g_6(t)$	0	0	0	0	0	0	0

2.3. Transition Probabilities and Mean Sojourn Times

Transition probabilities q_{ij} from state i to state j (Medhi, 2012) are given by the following equations:

$q_{01}(t) = ƛe^{-ƛt}$
$q_{12}(t) = p_1 \alpha e^{-\alpha t}$
$q_{13}(t) = p_2 \alpha e^{-\alpha t}$
$q_{14}(t) = p_3 \alpha e^{-\alpha t}$
$q_{15}(t) = p_4 \alpha e^{-\alpha t}$
$q_{16}(t) = p_5 \alpha e^{-\alpha t}$
$q_{17}(t) = p_6 \alpha e^{-\alpha t}$
$q_{20}(t) = g_1(t)$
$q_{30}(t) = g_2(t)$
$q_{40}(t) = g_3(t)$
$q_{50}(t) = g_4(t)$
$q_{60}(t) = g_5(t)$
$q_{70}(t) = g_6(t)$ (1-13)

The non-zero elements p_{ij} are given by the following equations:

$p_{01} = 1$
$p_{12} = p_1$
$P_{13} = p_2$
$p_{14} = p_3$
$p_{15} = p_4$
$p_{16} = p_5$
$p_{17} = p_6$
$p_{10} = g_1^*(0)$
$p_{20} = g_1^*(0)$
$p_{30} = g_2^*(0)$
$p_{40} = g_3^*(0)$
$p_{50} = g_4^*(0)$
$p_{60} = g_5^*(0)$
$p_{70} = g_6^*(0)$ (14-27)

From transition probabilities, the following can be verified easily:

$p_{12} + p_{13} + p_{14} + p_{15} + p_{16} + p_{17} = 1$
$p_{20} = p_{30} = p_{40} = p_{50} = p_{60} = p_{70} = 1$ (28-29)

The mean sojourn time μ_i is defined as the time of stay in the regenerative state i before transition to another state. If T denotes the sojourn time in the regenerative state i, then:

$$\mu_i = E(T) = \int_0^\infty P(T > t)dt \quad (30)$$

Hence, the mean sojourn time in the regenerative state 0 is given as:

$$u_0 = \int_0^\infty e^{-ƛt}\,dt = \frac{1}{ƛ} \quad (31)$$

The mean sojourn time in the regenerative sate 1 is given as:

$$u_1 = \int_0^\infty e^{-\alpha t}\,dt = \frac{1}{\alpha} \quad (32)$$

The unconditional time taken by the system to transit to any regenerative state j when the time is counted from the epoch of entrance into the state is derived mathematically using the following equation:

$$m_{ij} = \int_0^\infty t\,dQ_{ij} = -q_{ij}^{*\prime}(t) \quad (33)$$

Using the definition of unconditional time, the following equations are derived:

$m_{01} = \frac{1}{ƛ} = u_0$
$m_{12} = \frac{p_1}{\alpha}$
$m_{13} = \frac{p_2}{\alpha}$
$m_{14} = \frac{p_3}{\alpha}$
$m_{15} = \frac{p_4}{\alpha}$
$m_{16} = \frac{p_5}{\alpha}$
$m_{17} = \frac{p_6}{\alpha}$ (34-40)

The following relations can be easily verified:

$m_{12} + m_{13} + m_{14} + m_{15} + m_{16} + m_{17} = u_1$
$m_{20} = \frac{1}{\alpha_1} = u_2$
$m_{30} = \frac{1}{\alpha_2} = u_3$
$m_{40} = \frac{1}{\alpha_3} = u_4$
$m_{50} = \frac{1}{\alpha_4} = u_5$
$m_{60} = \frac{1}{\alpha_5} = u_6$
$m_{70} = \frac{1}{\alpha_6} = u_7$ (41-47)

Table 3 presents the estimated values of various rates for the system and Table 4 summarizes the estimate of various probabilities for each failure type.

Table 3. Estimated values of rates for the system

Rate (per hour)	Estimated value (per hour)
Failure rate, ƛ	8.837573E-06
Inspection rate, α	0.178461538
Repair rate for bushing failure, α_1	0.0210482
Repair rate for cooling system failure, α_2	0.0341880341
Repair rate for winding failure, α_3	0.0870322019
Repair rate for OLTC failure, α_4	0.0360750360
Repair rate for tank failure, α_5	0.06811989100
Repair rate for unidentified failure, α_6	0.03144654088

Table 4. Estimated probability values for each failure type

Probability	Estimated value
p_1	0.275862
p_2	0.172414
p_3	0.068966
p_4	0.103448
p_5	0.172414
p_6	0.206897

3. Mathematical Analysis

3.1. Mean Time Between Failures

Using simple probabilities arguments and the definition of $\emptyset_i(t)$, we get:

$$\emptyset_0(t) = Q_{01}(t) \quad (48)$$

Taking Laplace Stieltje's transform of the above equation and solving for $\emptyset_0^{**}(s)$, we obtain:

$$\emptyset_0^{**}(s) = Q_{01}^{**}(s) = \frac{N(s)}{D(s)} \quad (49)$$

MTBF when the system started at the beginning of state 0 is given as:

$$MTBF = \lim_{s\to 0} \frac{1-\emptyset_0^{**}(s)}{s} = \frac{N}{D} \quad (50)$$

where:

$N = u_0$

$$D = 1 \quad (51\text{-}52)$$

3.2. Availability Analysis

Using simple probabilities arguments and the definition of $A_i(t)$, we get:

$A_0(t) = M_0(t) + q_{01}©A_1(t)$

$A_1(t) = q_{12}©A_2(t) + q_{13}©A_3(t) + q_{14}©A_4(t) + q_{15}©A_5(t) + q_{16}©A_6(t) + q_{17}©A_7(t)$

$A_2(t) = q_{20}©A_0(t)$

$A_3(t) = q_{30}©A_0(t)$

$A_4(t) = q_{40}©A_0(t)$

$A_5(t) = q_{50}©A_0(t)$

$A_6(t) = q_{60}©A_0(t)$

$A_7(t) = q_{70}©A_0(t)$ (53-60)

Where,

$$M_0(t) = e^{-\lambda t} \quad (61)$$

Taking the Laplace transform of the above equations and solving for $A_0{}^*(s)$, we get:

$$A_0{}^*(s) = \frac{N_1(s)}{D_1(s)} \quad (62)$$

In steady state, the availability of the system is given by:

$$A_0 = \lim_{s\to 0} s\,A_0{}^*(s) = \frac{N_1}{D_1} \quad (63)$$

where,

$N_1 = u_0$

$D_1 = u_0 + p_1u_1 + p_2u_2 + p_3u_3 + p_4u_4 + p_5u_5 + p_6u_6$ (64-65)

3.3. Busy Period Analysis

Using simple probabilities arguments and the definition of $B_i(t)$, we get:

$B_0(t) = q_{01}©B_1(t)$

$B_1(t) = W_1(t) + q_{12}©B_2(t) + q_{13}©B_3(t) + q_{14}©B_4(t) + q_{15}©B_5(t) + q_{16}©B_6(t) + q_{17}©B_7(t)$

$B_2(t) = q_{20}©B_0(t)$

$B_3(t) = q_{30}©B_0(t)$

$B_4(t) = q_{40}©B_0(t)$

$B_5(t) = q_{50}©B_0(t)$

$B_6(t) = q_{60}©B_0(t)$

$B_7(t) = q_{70}©B_0(t)$ (66-73)

where,

$$W_0(t) = \bar{I}(t)$$

Taking the Laplace transform of the above equations and solving for $B_0{}^*(s)$, we get:

$$B_0{}^*(s) = \frac{N_2(s)}{D_2(s)} \quad (74)$$

In steady state, the expected busy period of the repairman is given by:

$$B_0 = \lim_{s\to 0} s\,B_0{}^*(s) = \frac{N_2}{D_2} \quad (75)$$

Where,

$N_2 = W_0^*(s)$

$D_2 = u_0 + P_1u_1 + P_2u_2 + P_3u_3 + P_4u_4 + P_5u_5 + P_6u_6$ (76-77)

3.4. Estimation of Reliability Indices

As a particular case, consider that the repair times are exponentially distributed, i.e.

$g_1(t) = \alpha_1 e^{-\alpha_1 t}$

$g_2(t) = \alpha_2 e^{-\alpha_2 t}$

$g_3(t) = \alpha_3 e^{-\alpha_3 t}$

$g_4(t) = \alpha_4 e^{-\alpha_4 t}$

$g_5(t) = \alpha_5 e^{-\alpha_5 t}$

$g_6(t) = \alpha_6 e^{-\alpha_6 t}$ (78-83)

Using the values given in Tables 3 and 4 and the expressions obtained in Section 3, the following reliability indices of the system are obtained:

Mean time between failures = 113153 hours

Availability of the system = 0.99972

Expected busy period of the repairman = 8.84E-06

4. Sensitivity Analysis

Sensitivity analysis is an approach used to assess the impact of a parameter on the derived reliability measures while holding the other parameters as constants. Relative sensitivity analysis is further established by standarizing the sensitivity analysis. Tables 5, 6 and 7 show the results of sensitivity analysis and relative sensitivity analysis conducted for MTBF, availability, and busy period.

Table 5. Sensitivity analysis for MTBF

Parameter (r)	Sensitivity analysis $dM = \frac{\partial(MTBF)}{\partial r}$	Relative Sensitivity Analysis $\delta M = \frac{dM}{MTBF} \times r$
λ	-1.28E+10	-1.000
α	0	0
α_1	0	0
α_2	0	0
α_3	0	0
α_4	0	0
α_5	0	0
α_6	0	0

Table 6. Sensitivity analysis for availability

Parameter (r)	Sensitivity analysis $dA = \frac{\partial(A_0)}{\partial r}$	Relative sensitivity analysis $\delta A = \frac{dA}{\partial A_0} \times r$
λ	-30.9028	-0.00027318
α	0	0
α_1	0.0055	0.00011579700
α_2	0.0013	0.0000444565
α_3	0.0000804	0.00000700113
α_4	0.000702	0.0000253356
α_5	0.000328	0.0000223624
α_6	0.0018	0.0000566192

Table 7. Sensitivity analysis for the busy period

Parameter (r)	Sensitivity analysis $dB = \frac{\partial(B_0)}{\partial r}$	Relative sensitivity analysis $\delta B = \frac{dB}{B_0} \times r$
λ	5.6004	0.999735027
α	-0.000277	-1.000000308
α_1	0.00000272	0.000115795
α_2	0.000000064	0.0000445569
α_3	0.0000000009	0.00000700115
α_4	0.0000000348	0.0000253357
α_5	0.0000000163	0.0000223622
α_6	0.0000000915	0.0000581298

Table 8. Life data analysis

	Estimated	Lower Bound	Upper Bound
Conditional reliability	0.996687	0.995125	0.997749
Conditional failure probability after 200 hrs	0.003313	0.002251	0.004875
Reliable life	32895.50	24205.43	44705.40
BX% life	27911.54 hrs	19961.08	39028.65
Mean life	82488.48 hrs	62844.92	108272.05
Mean remaining life	76996.70 hrs	57529.39	103051.53
Failure rate	1.131714E-07 per hr	2.90E-09	0.000004

Table 9. Summary of failure rate and reliability for each type of failure

	Failure rate (per hour)	Mean Time between failures (hours)	Reliability
Bushing failure	0.000013	78513	0.932345
Cooling system failure	0.000013	77799	0.931476
Winding failure	0.000008	123218.283	0.956345
OLTC failure	0.000013	77278.754	0.931303
Tank failure	0.000012	82028.866	0.935149
Unidentified failure	0.000013	78964.02313	0.932718

Tables 5, 6 and 7 show that the failure rate has a negative impact on the MTBF, whereas the availability of the system is not significantly influenced by any of the reliability parameters. However, the busy period is negatively influenced by the inspection rate and positively influenced by the failure rate.

5. Statistical Analysis

Table 8 summarizes the life data analysis of the power transformer system, assuming that the exponential distribution is the most appropriate distribution for the failure times data.

Table 9 presents the failure rate and reliability of the power transformer system based on the failure types.

The following can be deduced from Tables 8 and 9:

- There is a 99% chance that the transformer will operate successfully after 55000 hours of operation. The chance that the transformer will fail to operate after 200 hours, given that it was in operation for 55000 hours, is 0.033%.
- The average time that the transformer is expected to operate before failure is 32895.50 hours.
- The mean remaining lifetime for the transformer is 76996.70 hours. The 95% confidence bound is (57529.397, 103051.5303).
- The mean time between failures of each type is approximately similar except for the winding failure, for which it is 123218.283 hours.

6. Conclusion and Future Work

A probabilistic analysis of the power transformers in a power distribution company has been conducted in this paper, considering six failure modes and inspection.

Reliability indices, namely mean time between failures (113153 hours), availability (0.99972), and expected busy period of the repairman (8.84E-06) have been obtained to assess the transformers' effectiveness. These values give a positive indication of the operational capabilities of the transformers.

The results of sensitivity analysis and relative sensitivity analysis reveal that the mean time between transformers' failures is highly influenced negatively by the failure rate. At the same time, the reliability parameters have no significant influence on the availability of the transformers.

However, the expected busy period of the repairman is highly influenced positively by the failure rate and negatively by the inspection rate. Statistical analysis further reveals that the meantime for the transformers to operate before failure is 82488.48 hours, with a reliability of 99% after 55000 hours of operation and a reliable life of 32895.50 hours at a 95% reliability level. Also, the mean time between transformers' failures is approximately the same for each failure type, except in the case of winding failure, as the failure rate in this case is very small.

The failure-maintenance data used for this study has been collected from a company located in the southern region of Oman called Dhofar, where the weather is rainy and humid during the period from June to September popularly known as Khareef season.

During the Khareef season, the number of tourists in Dhofar is dramatically high, which increases the load on the power distribution network and can cause interruptions. Hence, there is a potential to extend this work for studying the performance of the system during the Khareef season to revising the existing maintenance strategies in order to reduce the number of breakdowns and hence increase the availability.

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Appendix

Acronyms

MTBF	Mean time between failures.
OLTC	On load tap changer.
Pdf	Probability density function.
Cdf	Cumulative distribution function.
S_i	State i.
λ	Constant failure rate.
p_1	Probability of bushing failure.
p_2	Probability of cooling system failure.
p_3	Probability of winding failure.
p_4	Probability of OLTC failure.
p_5	Probability of tank failure.
p_6	Probability of unidentified failure.
α	Inspection rate.
α_1	Repair rate for bushing failure.
α_2	Repair rate for cooling system failure.
α_3	Repair rate for winding failure.
α_4	Repair rate for OLTC failure.
α_5	Repair rate for tank failure.
α_6	Repair rate for unidentified failure.
$i(t), I(t)$	pdf and cdf of inspection time for the failed unit, respectively.
$g_1(t), G_1(t)$	pdf and cdf of repair times for bushing failure, respectively.
$g_2(t), G_2(t)$	pdf and cdf of repair times for cooling system failure, respectively.
$g_3(t), G_3(t)$	pdf and cdf of repair times for winding failure, respectively.
$g_4(t), G_4(t)$	pdf and cdf of repair times for OLTC failure, respectively.
$g_5(t), G_5(t)$	pdf and cdf of repair times for tank failure, respectively.
$g_6(t), G_6(t)$	pdf and cdf of repair times for undefined failure, respectively.
q_{ij}	pdf from state i to state j.
Q_{ij}	cdf from state i to state j.
©	Laplace convolution.
Ⓢ	Laplace Stieltje's convolution.
*	Laplace transform.
**	Laplace Stieltje's transform.
$\emptyset_i(t)$	cdf of first passage time from the regenerative state i to a failed state.
$A_i(t)$	Probability that the system is in upstate at instant t given that the system entered the regenerative state i at time t=0.
$B_i(t)$	Probability that the repairman is busy at instant t given that the system entered the regenerative state i at time t=0.