Performance Improvement in OFDM System Using Modified Sinc Pulse as Receiver Window

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Abstract- The Orthogonal Frequency Division Multiplexing (OFDM) system have some problems such as carrier frequency offset, timing offset, and Peak to Average Power Ratio (PAPR). The impacts of these problems results into inter carrier interference (ICI) which affects the bit error rate (BER). Therefore, suitable methods are required to mitigate ICI at the receiver. In this paper a new pulse shape, by modifying Sinc Pulse (SP) has been introduced, and used in OFDM system at the receiver as windowing in order to reduce ICI power. The pulse shape has been designed to obtain a small average ICI power and better BER performance. It has been observed that the proposed pulse window has better SIR performance than that of ISP pulse window at the normalized frequency offset is less than 0.06. The BER performance of proposed window has been compared with that of conventional Nyquist windows and MBH window. The proposed

Index Terms – OFDM, ICI, Receiver windowing, Frequency offset, BER, Sinc Pulse, ISP.

I. INTRODUCTION

The broadband wireless communication system demand is growing with a rapid pace. These systems are required to be operating in an environment which is characterized by high carrier frequency, data transmission rate and mobility. Such an environment can be modeled by a frequency selective fast time varying fading channel. To reduce the effect of multipath fading channel and to accommodate as much as number of users, the parallel transmission with multicarrier has been evolved. OFDM signals are more robust to frequency selective fading and easy to generate in comparison to all other multicarrier signals.

Besides several advantageous features, the OFDM systems also have some major problems those must be resolved. The impact of carrier frequency offset, timing offset result into generation of ICI. Therefore, suitable methods are required to mitigate ICI at the receiver. Several methods of ICI reduction are available [1]–[4]. Among them receiver windowing gains so much popularity due to its easy implementation and better performance. For receiver windowing, a Nyquist pulse shape with low side lobe level is required. Several pulse shapes are used for ICI reduction [1]. There is a scope for improving the performance of the OFDM system using modified sinc pulse at receiver windowing.

In this paper, an ICI analysis for receiver windowing OFDM systems operating in a Rayleigh fading channel, when carrier frequency offset exists, has been provided. Several robust frequency offset independent windowing Nyquist shapes, including the RC window, BTRC window, rectangular window, MBH window [5], and Improved Sinc Pulse (ISP) [2] have been considered for comparison. The effect of modified Sinc pulse, as receiver windowing function, on ICI reduction has been studied taking SIR and BER as the performance index.

The paper is organized as follows. In Section II, the system model is given. Analysis of ISI and SIR are included in section III. The proposed window function is given in Section IV. The effects of different window functions on the ICI, SIR and BER are compared in section V. The conclusions are presented in section VI.

II. SYSTEM DESCRIPTION

The discrete time base band OFDM system model with receiver windowing is shown in Fig.1. It consists of transmitter, channel and receiver with windowing blocks which are described below.

At the transmitter, complex data symbols \( \{ X(k) \} \) for \( k = 0, 1, \ldots, N - 1 \) are obtained by encoding input bits using modulation techniques like M-PSK, M-QAM, etc, and a block of \( N \) complex data symbols are converted from serial to parallel. These parallel data symbols are then modulated by a group of orthogonal subcarriers which are generated from IFFT block. The subcarriers satisfy the orthogonality as following.

\[
\frac{1}{T_u} \int_{0}^{T_u} e^{j2\pi f_k t} e^{j2\pi f_m t} dt = \begin{cases} 0 & \text{where } m \neq k \\ 1 & \text{where } m = k \end{cases}
\]

(1)

where, \( f_k = \frac{k}{T} \), \( f_k = 0, 1, 2, \ldots, N - 1 \) and \( \frac{1}{T} \) is the minimum spacing required. The base band OFDM signal transmitted during \( k \)th block can be written as

\[
x(i, t) = \frac{1}{N} \sum_{k=0}^{N-1} X(i, k)e^{j2\pi f_k t}, \quad \text{for } 0 \leq t \leq T_u
\]

(2)
where, $T_u (= NT_s)$ is the useful period of one OFDM symbol, $T_s$ is the sampling interval, $f_k (= \frac{k}{T_s})$ is the subcarrier frequency of the $k^{th}$ subcarrier, $X[i,k]$ is the complex data symbol of $i^{th}$ block, modulated on $k^{th}$ subcarrier, $N$ is the total number of subcarriers. The discrete version of the base band OFDM signal $x[i,n]$ can be expressed as

$$x[i,n] = \frac{1}{N} \sum_{k=0}^{N-1} X[i,k] e^{i \frac{2\pi kn}{N}}, \text{ for } n = 0, 1, 2, \ldots N-1.$$  

To avoid Inter Symbol Interference (ISI) a guard band is inserted between adjacent OFDM symbols. But a sudden change of signal contains high spectral components that results in ICI. So the guard band insertion with cyclic prefix (CP) is preferred. Due to CP insertion, the transmitted signal is extended to $T_{sym} = T_s + T_g$ and the discrete version of the base band OFDM signal with CP can be expressed as

$$x[i,n] = \begin{cases} x(i, n + N), & \text{for } n = 0, 1, \ldots, G-1 \\ \frac{1}{N} \sum_{k=0}^{N-1} X(i, k) e^{i \frac{2\pi kn}{N + G}} , & \text{for } n = G, G+1, \ldots, L-1 \end{cases}$$  

where, $N$ is total number of subcarriers and $G$ is total number of CP samples appended in an OFDM symbol during $i^{th}$ block transmission.

Generally OFDM system is used in wireless environment, where the multipath fading has much significance. Therefore multipath fading channel has to be considered for this study. The delay line model with L path is considered for frequency selective fading channel. The impulse response $h[l,\tau, t]$ for this channel, as given [6] is

$$h[l,\tau, t] = \sum_{l=0}^{L-1} h[l(t)] \delta(t - \tau_l)$$  

where, $h[l(t)]$ is a tap coefficient and $\tau_l$ is a propagation delay of the $l^{th}$ path, respectively. The tap coefficients $h[l(t)] (l = 1, 2, 3, \ldots, L - 1)$ are modeled as zero mean complex Gaussian random variables having variances $\sigma_l^2$ with $\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots + \sigma_L^2 = 1$.

The $j^{th}$ received signal, after considering the effect of multipath fading channel $h[l,\tau, t]$, is

$$r[i,n] = \sum_{l=0}^{L-1} h[l](n) \tilde{x}(i, n - \tau_l) + w(i, n),$$  

where, $w[i,n]$ represents the additive white Gaussian noise (AWGN) at the receiver with power spectral density of $N_0/2$.

A receiver model with time domain windowing is considered in Fig. 1. Receiver with time domain windowing uses ISI-free part of guard interval for windowing [7][8]. The samples of ISI-free duration $T_g$ together with N-samples of usable period $T_s$ are symmetrically multiplied with window [9]. After that, zero padding is done to make total number of samples to $2N$. Subsequently, the 2N point FFT is calculated for the resultant signal. The $i^{th}$ sample of $j^{th}$ OFDM symbol, after passing through a multipath fading and AWGN channel, in the presence of CFO ($\Delta f$) can be expressed as

$$r[i,n] = e^{i \frac{2\pi mn}{N}} \sum_{l=0}^{L-1} h[l](n) \tilde{x}(i, n - \tau_l) + w(i, n).$$

![Fig. 1: Base band OFDM system model](image-url)
where, $\epsilon$ is the CFO normalized by the subcarrier spacing 
\[
\frac{1}{T_c} = \frac{1}{N T_c}.
\]
$h_i(\tau)$ is response of frequency selective multipath fading channel. For time domain windowing, only $V - N$ samples of received signal are taken
\[
r(i, n + G - V) = \sum_{i=-\infty}^{L-1} h_i(\tau) w(i, n + G - V - \tau)e^{j2\pi(n+G-V-V)} + w(i, n + G - V), \quad for \quad n = 0, 1, 2, \ldots, V + N - 1
\]
(8)

Now, the signal $r(i, n + G - V)$ is symmetrically multiplied with the window function $w(n)$ and then, the resultant signal $y(i, n) = r(i, n + G - V) \times w(n)$ is extended to $2N$ points by inserting zeros at both the sides symmetrically.
\[
y(i, n) = \begin{cases} 
0, & 0 \leq n < \frac{N(1-\alpha)}{2} \\
y(i, n), & \frac{N(1-\alpha)}{2} \leq n < \frac{3N(1+\alpha)}{2} \\
0, & \frac{3N(1+\alpha)}{2} \leq n < 2N - 1 
\end{cases}
\]
(9)

where, $\alpha = V/N$ is a roll-off factor.

The $2N$-point FFT of resultant signal $\tilde{y}(i, n)$ is then taken, which is denoted by $\tilde{X}(i, \rho)$
\[
\tilde{X}(i, \rho) = \sum_{n=0}^{2N-1} \tilde{y}(i, n)e^{-j2\pi mn}, \quad for \quad p = 0, 1, \ldots, 2N
\]
(10)

After simplification, it becomes
\[
\tilde{X}(i, \rho) = \sum_{k=0}^{N-1} X(i, k)H(k, \Delta f)S(k, p, \Delta f) + W(i, \rho), \quad \text{for} \quad p = 0, 1, \ldots, 2N - 1
\]
(11)

where, $H(k)$ is frequency response of channel to the $k$th subcarrier [9] and $S(k, p, \Delta f)$ is the ICI coefficient [1], defined as
\[
S(k, p, \Delta f) = \frac{1}{N}e^{-j2\pi kp/N}e^{j2\pi (k+p+2\alpha)} \sum_{n=0}^{2N-1} p(n)e^{j2\pi(2k-p+2\epsilon)}
\]
(12)

As it is known, that either even or odd terms contain the desired information; therefore the transmitted data sequence $X(i, q)$ for $q = 0, 1, \ldots, N - 1$ can be recover from $\tilde{X}(i, \rho)$ by choosing only even terms at $\rho = 2q$. That is
\[
X(i, 2q) = \sum_{k=0}^{N-1} X(i, k)H(k, \Delta f)S(k, 2q, \Delta f) + W(i, 2q), \quad \text{for} \quad q = 0, 1, \ldots, N - 1
\]
(13)

After breaking the summation in two terms
\[
X(i, 2q) = \sum_{k=0}^{N-1} X(i, k)H(k, \Delta f)S(k, 2q, \Delta f) + W(i, 2q)
\]
\[
+ X(q, q)H(q, \Delta f)S(q, 2q, \Delta f), \quad \text{for} \quad q = 0, 1, \ldots, N - 1
\]
(14)

The first term in equation (14) is the ICI component, the second term is the AWGN, and the last term is the desired signal.

III. ICI POWER AND SIR

To reduce this ICI term, it is necessary to analyze the ICI term in detail. Therefore, the expression of ICI power and SIR are given in this section. The ICI power can be determined from equation (13)
\[
\sigma_{ICl}^2 = \sum_{k1=0, k1 \neq k}^{N-1} E[X(i, k1)X^*(i, k2)]
\]
\[
E[(H(k1, \Delta f)S(k1, 2q, \Delta f)(H(k2, \Delta f)S(k2, 2q, \Delta f))]^*
\]
(15)

\[
\sigma_{ICl}^2 = \frac{1}{N} \sum_{k1=0}^{N-1} \sum_{n1=0}^{2N-1} \sum_{n2=0}^{2N-1} p(n1)p(n2) e^{j2\pi(\alpha(n1-n2)(k1-k2)+\epsilon)}
\]
(16)

On rearranging the above equation and substituting $\sigma_{ICl}^2 - 1[10]$ the ICI power in terms of Fourier transform can be written as
\[
\frac{\sigma_{ICl}^2}{\sigma_{ICl}^2} = \sum_{k=0, k \neq q}^{N-1} |P(k - q + \epsilon)|^2
\]
(17)

The Power of desired signal is
\[
\sigma_{D}^2 = E[|X(i, q)|^2]E[|H(q, \Delta f)S(q, 2q, \Delta f)|^2] = |P(e)|^2
\]
(18)

Now, signal to interference ratio $SIR_e$ is defined as the ratio of the average desired signal power to the average ICI power.
\[
SIR_e = \frac{\sigma_{D}^2}{\sigma_{ICl}^2} = \frac{1}{\sum_{k=0, k \neq q}^{N-1} |P(k - q + \epsilon)|^2}
\]
(19)

IV. PROPOSED WINDOW FUNCTION

It has been shown in the last section that the ICI power depends on the Fourier transform of window function used at the receiver. Several window functions such as rectangular, RC, BTRC, Sinc pulse and Improved Sinc Pulse [11] have been used for ICI reduction. The proposed pulse is a modification of Sinc power pulse.

The expression of Sinc power pulse [12] is given as
An improved Sinc pulse (ISP), has been proposed by modifying Sinc pulse, provides a good improvement in SIR and BER compared with Sinc pulse \[13\][14]. The expression of ISP is given as

\[
P_{isp}(f) = Sinc^n(fT)
\]

(20)

where, \(n\) is a design parameter to adjust the amplitude, \(Sinc\) is degree of Sinc function.

Further improved performance with respect to the reduction of average ICI power of an N OFDM systems can be obtained by changing the factor in the expression of ISP pulse in frequency domain.

The new proposed window is obtained by replacing the exponential factor in equation (21), given as

\[
P(f) = \frac{1}{\rho} Sinc^2(fT_s)(erf(c+ f + d) - erf(c+ f - d))
\]

(22)

where, \(\rho\) and \(d\) are design parameters to adjust the amplitude, \(T_s\) is the duration of the OFDM symbol. The lower and upper bounds on \(c\) and \(d\) are determined by gradient search method and they are exactly confirmed by the simulation studies. The Modified Sinc pulse has been plotted along with Sinc pulse and ISP pulse, for different values of the parameters, in Fig.2. From the plot some points has to be concluded.

- The proposed window has the main lobe width as same as Sinc power pulse at the given design parameters, where as the ISP has less main lobe width.

- The first sidelobe level of the proposed pulse is very much less compared to Sinc pulse, and is even less than that of ISP pulse.

The proposed pulse compared with different values of design parameters in Fig.3 and the observations are:

- The parameters \(c\) and \(d\) are taken such that, they effect the mainlobe width and sidelobe levels.

- When \(d \ll 1\), the window has suppressed sidelobes, but the main lobe gets attenuated.

- When \(d = 1\), the mainlobe width is sufficient (same as Sinc pulse), and the sidelobe levels are very less.

- When \(d \gg 1\), the pulse is overlapped by Sinc pulse, i.e; there exists First sidelobe level, and the effect of error function given in equation (22) is very less.

The proposed window has been also plotted along with other Nyquist pulse shapes as shown in Fig.4. When it has compared with all other Nyquist pulse windows, it has less side lobe level and maintained the mainlobe width.

V. SIMULATION RESULTS

The proposed pulse shape has been studied in previous section. This pulse shape has been used as a window at the receiver of the OFDM system in order to reduce ICI. When there is a frequency offset the ICI power increases. So the evaluation of proposed window function, the ICI power for different values of frequency offset as given in equation (17) has been determined using MATLAB and plotted in dB as shown in Fig.5. In this figure the pulse shape parameters are chosen as the following; \(n = 2\), and \(a = 1\). The results show that for normalized frequency less than \(1.0\) the ICI power is minimum for modified Sinc pulse shape compared to ISP pulse shape.
When it has been compared with all other pulses it has better performance in terms of ICI than any other pulse. Fig.6 shows The SIR performance with respect to the normalized frequency offset. The result shows that for the normalized frequency offset less than 0.06, modified Sinc pulse shape is outperforming to all other pulse shapes, for all other range of normalized frequency offset ISP pulse shape has better performance.

To evaluate the BER performance of receiver windowing technique first the effect of frequency offset on BER performance is investigated using the OFDM model given in Fig.1. The different simulation parameters considered are as included in Table. I.

Table 1: OFDM system parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>FFT Size</td>
<td>64</td>
</tr>
<tr>
<td>Number of Carriers</td>
<td>64</td>
</tr>
<tr>
<td>Guard length</td>
<td>16</td>
</tr>
<tr>
<td>Symbol rate</td>
<td>25000</td>
</tr>
<tr>
<td>Signal constellation</td>
<td>16 QAM</td>
</tr>
<tr>
<td>Channel</td>
<td>Rayleigh Fading channel with AWGN</td>
</tr>
<tr>
<td>OFDM symbols for 1 loop</td>
<td>12</td>
</tr>
<tr>
<td>Number of simulation loops</td>
<td>1000</td>
</tr>
</tbody>
</table>

It has been found that as the normalized frequency increases the BER is also increasing. After performing the receiver windowing it has found that there is an improvement in BER performance.

The BER performance of OFDM system with receiver windowing has been compared with that of without windowing in Fig.7. For this evaluation pulse shaping parameters are selected as $n = 1, \alpha = 1, \frac{d}{r} = 1$, and the normalized frequency offset of 0.04 is considered.
The combined analysis of ICI and BER of proposed window family and comparison with other available pulse shapes are tabulated in Table 2.

### VI. Conclusion

Inter carrier Interference (ICI) is a major problem of OFDM system which effects the performance in terms of SIR and BER. Different methods of ICI reduction are investigated. The receiver time domain windowing is used to reduce the sensitivity to frequency errors. A number of pulse shaping functions are considered as window for ICI power reduction.

A modified Sinc pulse has been proposed as a new window function to improve the performance of OFDM system. The performance of each window function is evaluated and compared with each other using the parameters such as ICI power, SIR (Signal to Interference Ratio) and BER (Bit Error Rate). Simulation results show that proposed pulse shape and ISP pulse provides similar performance in terms of ICI power reduction, SIR. At lower range of normalized frequency offset the proposed pulse window outperforms than all other window functions. The BER performance of proposed window is better compared to conventional windows such as rectangular, RC and BTRC. When it compared with MBH window function, at low range of SNR the proposed window performs similar to that of MBH window, and has better performance at SNR more than 10 dB.

### VII. References