Soft Pre Generalized - Closed Sets in a Soft Topological Space

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Abstract

This paper introduces soft pre-generalized closed set in soft topological spaces. The notations of soft pre interior and soft pre closure are generalized using these sets. In a soft topological space, a soft set \( F_A \) is said to soft pre generalised –closed if \( p(F_A) \subseteq F_0 \) whenever \( F_A \subseteq F_0 \) where \( F_0 \) is a soft \( P \)-open set. A detail study is carried out on properties of soft Pre generalized closed sets and soft PT\(_{1/2}\) Space.

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Introduction. Soft set theory was first introduced by Molodtso\v{v} [5] in 1999 as a general mathematical tool for dealing uncertain fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. Modern topology depends strongly on the ideas of set theory. In 2010 Muhammad shabir, Munazza Naz [6] used soft sets to define a topology namely Soft topology. J.Subhashini and C.Sekar defined soft pre-open sets [11] in a soft topological space. In General topology the concept of generalized closed set was introduced by Levine [3] plays a significant. This notation has been studied extensively in recent years by many topologies. The investigation of generalized closed sets has led to several new and interesting concepts, e.g. new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. Soft generalised closed set was introduced by K.Kannan[2] in 2012. In this paper we introduce soft pre-generalized closed set and soft PT\(_{1/2}\) – Space in a soft topological space and a detailed study of some of its properties.

2. Preliminaries

For basic notations and definitions not given here, the reader can refer [1-12].

2.1. Definition. [11] A soft set \( F_A \) on the universe \( U \) is defined by the set of ordered pairs \( F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\} \), where \( E \) is a set of parameters, \( A \subseteq E \), \( P(U) \) is the power set of \( U \), and \( f_A : A \rightarrow P(U) \) such that \( f_A(e) = \emptyset \) if \( e \notin A \). Here, \( f_A \) is called an approximate function of the soft set \( F_A \). The value of \( f_A(e) \) may be arbitrary, some of them may be empty, some may have non-empty intersection. Note that the set of all soft set over \( U \) is denoted by \( S(U) \).
2.2. Example. Suppose that there are five cars in the universe. Let \( U=\{c_1, c_2, c_3, c_4, c_5\} \) under consideration, and that \( E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\} \) is a set of decision parameters. The \( e_i \) \((i = 1, 2, 3, 4, 5, 6, 7, 8)\) stand for the parameters “expensive”, “beautiful”, “manual gear”, “cheap”, “automatic gear”, “in good repair”, “in bad repair” and “costly” respectively. In this case, to define a soft set means to point out expensive cars, beautiful cars and so on. It means that, consider the mapping \( f_E \) given by “cars (.)”, where (.) is to be filled in by one of the parameters \( e_i \in E \). For instance, \( f_E(e_1) \) means “car (expensive)”, and its functional value is the set \( \{c \in U: c \text{ is an expensive car}\} \) and so,

Let \( A \subseteq E \), the soft set \( F_A \) that describes the “attractiveness of the cars” in the opinion of a buyer say Ram, may be defined like \( A=\{e_2, e_3, e_4, e_5, e_7\} \), \( f_A(e_2)=\{c_2, c_3, c_5\} \), \( f_A(e_3)=\{c_2, c_4\} \), \( f_A(e_4)=\{c_1\} \), \( f_A(e_5)=\{U\} \) and \( f_A(e_7)=\{c_1, c_3\} \). We can view this soft set \( F_A \) as consisting of the following collection of approximations:

\[
F_A = \{(e_2,\{c_2, c_3, c_5\}), (e_3,\{c_2, c_4\}), (e_4,\{c_1\}), (e_5,\{U\}), (e_7,\{c_1, c_3\})\}.
\]

2.3. Definition. [11] The soft set \( F_A \in S(U) \) is called a soft point in \( F_E \), denoted by \( (e,f_A) \), if for the element \( e \in A \) and \( f_A(e) \neq \emptyset \) and \( f_A(e') = \emptyset \) for all \( e' \in A - \{e\} \). The soft point \( (e,f_A) \) is said to be in the soft set \( F_B \), denoted by \( (e,f_A) \in F_B \) if for the element \( e \in A \) and \( f_A(e) \subseteq f_B(e) \).

2.4. Definition. [10] Let \( F_A \in S(U) \). The soft power set of \( F_A \) is defined by \( \bar{P}(F_A) = \{F_A: F_A \subseteq F_{A_i}, i \in I \subseteq N\} \) and its cardinality is defined by \( |\bar{P}(F_A)| = 2^{\sum_{i=1}^{n}f_A(e)|} \), where \( f_A(e) \) is the cardinality of \( f_A(e) \).

2.5. Example. [10] Let \( U=\{u_1, u_2\} \), \( E=\{e_1, e_2, e_3\} \), \( A \subseteq E \) \( A=\{e_1, e_2\} \) and \( F_A = \{(e_1,\{h_1, h_2\}), (e_2,\{h_1, h_2\})\} \). Then \( F_{A_1}=F_A \), \( F_{A_2}=F_A \), \( F_{A_3}=(e_1,\{h_1\}), F_{A_4}=(e_1,\{h_1\}), F_{A_5}=(e_1,\{h_1\}), F_{A_6}=(e_1,\{h_1\}), F_{A_7}=(e_1,\{h_1\}), F_{A_8}=(e_1,\{h_1\}), F_{A_9}=(e_1,\{h_1\}), F_{A_{10}}=(e_1,\{h_1\}), F_{A_{11}}=(e_2,\{h_1\}), F_{A_{12}}=(e_2,\{h_1\}), F_{A_{13}}=(e_2,\{h_1\}), F_{A_{14}}=(e_2,\{h_1\}), F_{A_{15}}=(e_2,\{h_1\}), F_{A_{16}}=(e_2,\{h_1\}) \) are all soft subset of \( F_A \). So \( |\bar{P}(F_A)| = 2^4 = 16 \).

2.6. Definition. [11] Let \( F_E \in S(U) \). If \( F_E(e) = U \) for all \( e \in E \), then \( F_E \) is called a soft universal set (soft absolute set) denoted by \( F_E \) or \( \bar{U} \).

2.7. Definition. [6] Let \( F_E \in S(U) \). A soft topology on \( F_E \) denoted by \( \tau \) is a collection of soft subsets of \( F_E \) having the following properties:

(i). \( F_\emptyset, F_E \in \tau \)

(ii). \( \{F_{E_i}: i \in I \subseteq N\} \subseteq \tau \Rightarrow \bigcup_{i \in I} F_{E_i} \in \tau \)

(iii). \( \{F_{E_i}: 1 \leq i \leq n, n \in N\} \subseteq \tau \Rightarrow \bigcap_{i=1}^{n} F_{E_i} \in \tau \).
The pair \((F_E, \bar{\tau})\) is called a soft topological space.

2.8. Example. Let us consider the soft subsets of \(F_A\) that are given in Example 2.5. Then \(\bar{\tau}_1 = \{F_\emptyset, F_A, F_{A_3}, F_{A_4}\}\),
\(\bar{\tau}_2 = \{F_\emptyset, F_A, F_{A_3}\}\), \(\bar{\tau}_3 = \{\bar{\alpha} (F_A)\}\) are soft topologies on \(F_A\).

2.9. Definition. [5] Let \((F_E, \bar{\tau})\) be a soft topological space. Then, every element of \(\bar{\tau}\) is called a soft open set. Clearly \(F_\emptyset\) and \(F_E\) are soft open sets. The collection of all soft open set is denoted by \(G_s(F_E)\). Let \(F_C \subseteq F_E\). Then \(F_C\) is said to be soft closed if the soft set \(F_C^c\) is soft open in \(F_E\). The collection of all soft closed set is denoted by \(F_c(F_E)\).

2.10. Definition. A soft topology whose soft open sets are all soft subsets of \(F_E\) is called soft discrete topology.

2.11. Definition. [1] Let \(F_E\) be a soft set over \(U\) and \(x \in U\), we say that \(x \in F_E\) read as \(x\) belongs to the soft set \(F_E\) whenever \(x \in f_E(e)\) for all \(e \in E\). Not that for any \(x \in U\), \(x \in F_E\) if \(x \notin f_E(e)\) for some \(e \in E\).

2.12. Definition. [1] The soft set \(F_E\) over \(U\) such that \(f_E(e) = \{x\}\) for all \(e \in E\) is called soft singleton and is denoted by \(x_E\).

2.13. Definition. [5] Let \((F_E, \bar{\tau})\) be a soft topological space over \(U\), \(F_E \subseteq S(U)\) and Let \(V\) be a non null subset of \(U\).
Then the soft subset of \(F_E\) over \(V\) denoted by \(vF_E\), is defined as follows: \(f_E(e) = V \cap f_E(e)\), for all \(e \in E\). In other words \(vF_E = \bar{V} \cap F_E\).

2.14. Definition. [9] Let \((F_E, \bar{\tau})\) be a soft topological space over \(U\). A soft set \(F_E\) is called a soft generalized closed in \(U\) if \(\bar{F}_E \subseteq F_O\) whenever \(F_E \subseteq F_O\) and \(F_O\) is soft open in \(U\).

2.15. Definition. [12] Let \((F_E, \bar{\tau})\) be a soft topological space, a soft set \(F_A\) is said to be soft pre-open set (soft P-open) if there exists a soft open set \(F_O\) such that \(F_A \subseteq F_O \subseteq F_A^c\). The set of all soft P-open set of \(F_E\) is denoted by \(G_{sp}(F_E, \bar{\tau})\) or \(G_{sp}(F_E)\). Then \(F_E^c\) is said to be soft pre-closed. The set of all soft P-closed set of \(F_E\) is denoted by \(F_{sp}(F_E, \bar{\tau})\) or \(F_{sp}(F_E)\).

2.16. Remark. [11] A soft set \(F_A\) which is both soft P-open and soft P-closed is known as soft P-clopen set. Clearly \(F_\emptyset\) and \(F_E\) are soft P-clopen sets.

2.17. Proposition. [11]
(i) Every soft open set is a soft pre-open set.
(ii) Every soft closed set is a soft pre-closed set.

2.18. Theorem. [10]
(i) Arbitrary soft union of soft P-open sets is a soft P-open set.
(ii) The soft intersection of any two soft P-open set need not be a soft P-open set.
(iii) Arbitrary soft intersection of soft P-closed sets is soft P-closed set.
(iv) The soft union of any two soft P-closed set need not be a soft P-closed set.
2.19. Definition. [11] Let \((F_E, \tilde{A})\) be a soft topological space and \(F_A \subseteq F_E\). Then the soft pre-interior (soft P-interior) of \(F_A\) denoted by \(p(F_A)^o\) is defined as the soft union of all soft P-open subsets of \(F_A\). Note that \(p(F_A)^o\) is the biggest soft P-open set that contained in \(F_A\).

2.20. Definition. [11] Let \((F_E, \tilde{A})\) be a soft topological space and \(F_A \subseteq F_E\). Then the soft pre closure (soft P-closure) of \(F_A\) denoted by \(p(F_A)\) is defined as the soft intersection of all soft P-closed supersets of \(F_A\). Note that, \(p(F_A)\) is the smallest soft P-closed set that containing \(F_A\).

2.21. Proposition. [11] A soft set is soft P-open iff \(p(F_A)^o = F_A\).

A soft set is soft P-closed iff \(p(F_A) = F_A\).

2.22. Definition. [3] A subset \(B\) of a topological space \(X\) is said to be:

(i) a generalized closed set if \(\tilde{B} \subseteq U\) whenever \(B \subseteq U\) and \(U\) is open in \(X\).

(ii) a pre generalized closed set if \(p(\tilde{B}) \subseteq U\) whenever \(B \subseteq U\) and \(U\) is pre-open in \(X\).

3. Soft P-generalized closed set

3.1 Definition. Let \(F_A \in S(U)\) is said to be soft pre generalized closed set (soft Pg-closed set) if \(p(F_A) \subseteq F_A\) whenever \(F_A \subseteq F_0\) and \(F_0 \in Gsp(F_E)\). The collection of all soft Pg-closed sets is denoted by \(Fpg(F_E)\).

3.2. Example. Let as consider the soft subsets in Example 2.5. Let \((F_A, \tilde{A})\) be a soft topological space where \(U = \{h_1, h_2\}\), \(A = \{e_1, e_2\}\), \(F_A = \{(e_1, \{h_1, h_2\}, \epsilon_2, \{h_1, h_2\}\}\) and \(\tilde{A} = \{F_0, F_A, A_3, A_4\}\), \(\tilde{A}^c = \{F_0, F_A, A_{10}, A_{11}\}\), \(Gsp(F_A) = \{F_0, F_A, A_4, A_5, A_6, A_{11}, A_{12}, A_{13}, A_{14}\}\), \(Fpg(F_A) = \{F_0, F_A, A_{10}, A_{12}, A_{11}, A_{12}, A_{11}, A_{12}, A_{11}\}\). Clearly for the soft subset \(A_{13}\), \(p(F_A) = A_{13} \subseteq A_{13}\) whenever \(A_{13} \subseteq A_{13}\) and \(A_{13} \subseteq Gsp(F_A)\) and for the soft subset \(A_{14}\), \(p(F_A) = A_{14} \subseteq A_{14}\) whenever \(A_{14} \subseteq A_{14}\) and \(A_{14} \subseteq Gsp(F_A)\). Therefore the soft subsets \(A_{13}\) and \(A_{14}\) are soft Pg-closed sets but the soft set \(A_{15}\), \(p(F_A) = A_{15}\) is not contained in any soft P-open set. Therefore \(A_{15}\) is not a soft Pg-closed set.

3.3. Proposition. Every soft P-closed set is a soft Pg-closed set.

Proof: Let \(F_C\) be a soft P-closed set, then for every soft P-closed set \(F_C\), \(F_C = p(F_C)\), we have every soft P-closed set is soft Pg-closed set. But the converse is not true in general.

3.4. Remark. A soft Pg-closed set need not be a soft P-closed set. The following example supports our claim.

3.5. Example. Let us consider the soft subsets in Example 2.5. Let \((F_A, \tilde{A})\) be a soft topological space where \(U = \{h_1, h_2\}\), \(A = \{e_1, e_2\}\), \(F_A = \{(e_1, \{h_1, h_2\}, \epsilon_2, \{h_1, h_2\}\}\) and \(\tilde{A} = \{F_0, F_A, A_3, A_4\}\), \(\tilde{A}^c = \{F_0, F_A, A_{10}, A_{11}\}\), \(Fpg(F_A) = \{F_0, F_A, A_4, A_5, A_6, A_{11}, A_{12}, A_{13}, A_{14}\}\). Then \(Fpg(F_A) = \{F_0, F_A, A_4, A_5, A_6, A_{11}, A_{12}, A_{13}, A_{14}\}\). Then by Example 3.2 soft subset \(A_{15}\) is a soft Pg-closed set but it is not a soft P-closed set.

3.6. Proposition.

(i) If \(F_A\) and \(F_B\) are soft pre generalized closed sets then so is \(F_A \tilde{\cap} F_B\).

(ii) Let \(F_A\) be a soft pre generalized closed set and suppose that \(F_B\) is a soft P-closed set. Then \(F_A \tilde{\cap} F_B\) is a soft pre generalized closed set.

3.7. Definition. Let \((F_E, \tilde{A})\) be a soft topological space and \(F_A \subseteq F_E\). Then the soft pre generalized closure (soft Pg-closure) of \(F_A\) denoted by \(pg(F_A)\) is defined as the soft intersection of all soft pre generalized closed supersets of \(F_A\).
3.8. Remark. Since the arbitrary soft intersection of soft pre generalized closed sets is a soft pre generalized closed set, \( pg(\overline{F_A}) \) is soft pre generalized closed set. Note that, \( pg(\overline{F_A}) \) is the smallest soft pre generalized closed set that containing \( F_A \).

3.9. Theorem. Let \((F_E, \bar{r})\) be a soft topological space and let \( F_A \) and \( F_B \) be a soft sets over \( U \). Then

(a) \( F_A \subseteq pg(\overline{F_A}) \)
(b) \( F_A \) is soft pre generalized closed iff \( F_A = pg(\overline{F_A}) \)
(c) \( F_A \subseteq F_B \) then \( pg(\overline{F_A}) \subseteq pg(\overline{F_B}) \)
(d) \( pg(\overline{F_A}) = F_B \) and \( pg(\overline{F_B}) = F_B \).
(e) \( pg(\overline{F_A \cap F_B}) \subseteq pg(\overline{F_A}) \cap pg(\overline{F_B}) \)
(f) \( pg(\overline{F_A \cup F_B}) = pg(\overline{F_A}) \cup pg(\overline{F_B}) \)
(g) \( pg(pg(\overline{F_A})) = pg(F_A) \)

Proof: Refer theorem 4.16 \(^{11}\)

3.10. Theorem. A soft subset \( F_A \) of a soft topological space \((F_E, \bar{r})\) is soft Pg-closed set iff \( p(\overline{F_A} \setminus F_A) \) does not contain any non-empty soft pre-closed set. (That is \( p(\overline{F_A} \setminus F_A) \) contains only null soft pre-closed set.)

Proof: Necessity. Suppose that \( F_A \) is soft Pg-closed set. Let \( F_C \) be a soft P-closed and \( F_C \subseteq p(\overline{F_A} \setminus F_A) \). Since \( F_C \) is soft P-closed, we have its relative complement \( F_C^c \) is soft P-open. Since \( F_C \subseteq p(\overline{F_A} \setminus F_A) \), we have \( F_C \subseteq p(\overline{F_A}) \) and \( F_C \subseteq F_A^c \). Hence \( F_A \subseteq F_C^c \). Consequently \( p(\overline{F_A}) \subseteq F_C^c \). [Since \( F_A \) is soft Pg-closed set]. Therefore, \( F_C \subseteq p(\overline{F_A}^c) \). Hence \( F_C = F_A \). Hence \( p(\overline{F_A} \setminus F_A) \) contains only null soft P-closed set.

 Sufficiency. Let \( F_A \subseteq F_O \), where \( F_O \) is soft pre open in \((F_E, \bar{r})\). If \( p(\overline{F_A}) \) is not contained in \( F_O \), then \( p(\overline{F_A}) \cap (F_E \setminus F_O) \neq F_O \). Now, since \( p(\overline{F_A}) \cap (F_E \setminus F_O) \subseteq p(\overline{F_A} \setminus F_A) \) and \( p(\overline{F_A}) \cap (F_E \setminus F_O) \) is a non-empty soft pre closed set, then we obtain a contradiction and therefore \( F_A \) is soft Pg-closed set.

3.11. Theorem (Sufficient condition for soft Pg-closed set is soft P-closed set.)

If \( F_A \) is a soft Pg-closed set of a soft topological space \((F_E, \bar{r})\), then the following are equivalent

(i) \( F_A \) is soft P-closed set.

(ii) \( p(\overline{F_A}) \setminus F_A \) is soft P-closed set.

Proof:

Let \((F_E, \bar{r})\) be a soft topological space. Let \( F_A \) be soft Pg-closed set.

(i) \( \rightarrow \) (ii)

Let \( F_A \) be soft P-closed set also it is a soft Pg-closed set, and then by Theorem 3.10, \( p(\overline{F_A} \setminus F_A) = F_O \) which is soft P-closed set.

(ii) \( \rightarrow \) (i)

Let \( p(\overline{F_A}) \setminus F_A \) be a soft P-closed set and \( F_A \) be soft Pg-closed. Then by Theorem 3.10, \( p(\overline{F_A}) \setminus F_A \) contains only null soft P-closed subset. Since, \( p(\overline{F_A}) \setminus F_A \) is soft P-closed and \( p(\overline{F_A}) \setminus F_A = F_O \). This shows that \( F_A \) is soft P-closed set.

3.12. Theorem. For a soft topological space \((F_E, \bar{r})\), the following are equivalent

(i) Every soft subset of \( F_E \) is soft Pg-closed set.

(ii) \( G_{sp}(F_E) = F_{sp}(F_E) \).
Proof: (i)→(ii)
Let \( F_0 \in G_{sp}(F_E) \). Then by hypothesis, \( F_0 \) is soft \( Pg \)-closed which implies that \( p(F_0) \subseteq F_0 \). So \( p(F_0) = F_0 \). Therefore \( F_0 \in F_{sp}(F_E) \). Also let \( F_C \in F_{sp}(F_E) \). Then \( F_C \subseteq F_{sp}(F_E) \), hence by hypothesis \( F_C \in F_{pg} \) is soft \( Pg \)-closed and then \( F_E \setminus F_C \in F_{sp}(F_E) \), thus \( F_C \in G_{sp}(F_E) \) according above we have \( G_{sp}(F_E) = F_{sp}(F_E) \).

(ii)→(i)
If \( A \) is a soft subset of a soft topological space \((F_E, \bar{\tau})\) such that \( F_A \subseteq F_0 \) where \( F_0 \in G_{sp}(F_E) \), then \( F_A \in G_{sp}(F_E) \) and therefore \( p(F_A) \subseteq F_0 \) which shows that \( F_A \) is soft \( Pg \)-closed set.

3.13. Theorem. If \( F_A \) is soft \( Pg \)-closed in \((F_E, \bar{\tau})\) and \( A \subseteq F_B \subseteq p(F_A) \), then \( F_B \) is soft \( Pg \)-closed set.

Proof: Suppose that \( F_A \) is soft \( Pg \)-closed set in \((F_E, \bar{\tau})\) and \( A \subseteq F_B \subseteq p(F_A) \). Let \( F_B \subseteq F_0 \) and \( F_0 \) is soft \( P \)-open in \((F_E, \bar{\tau})\). Since \( F_A \subseteq F_B \) and \( F_B \subseteq F_0 \), we have \( F_A \subseteq F_0 \). Hence \( p(F_A) \subseteq F_0 \) (since \( F_A \) is soft \( Pg \)-closed set). Since \( F_B \subseteq p(F_A) \), we have \( p(F_B) \subseteq p(F_A) \subseteq F_0 \). Therefore, \( F_B \) is soft \( Pg \)-closed set.

4. Soft pre generalized open set

4.1. Definition. Let \((F_E, \bar{\tau})\) be a soft topological space. A soft set \( F_A \) is called a soft pre generalized open set (soft \( Pg \)-open set) if the complement \( F_A^c \) is soft \( Pg \)-closed in \((F_E, \bar{\tau})\). The collection of all soft \( Pg \)-open sets is denoted by \( G_{spg}(F_E) \). Equivalently, a soft set \( F_A \) is called a soft \( Pg \)-open set in a soft topological space \((F_E, \bar{\tau})\) if and only if \( F_A \subseteq p(F_A)^o \) whenever \( F_C \subseteq F_A \) and \( F_C \) is soft \( P \)-closed set in \((F_E, \bar{\tau})\).

4.2. Example. Let as consider the soft subsets in Example 2.5. Let \((F_A, \bar{\tau})\) be a soft topological space where \( U = \{h_1, h_2\} \), \( A = \{e_1, e_2\} \), \( F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\} \) and \( \bar{\tau} = \{F_0, F_A, F_{A_0}, F_{A_2}\} \). \( F_{sp}(F_E) = \{F_0, F_A, F_{A_0}, F_{A_2}, F_{A_4}, F_{A_6}, F_{A_8}, F_{A_{12}}, F_{A_{14}}, F_{A_{16}}\} \). Clearly for the soft subset \( F_{A_0} \), \( F_0 \subseteq p(F_{A_0})^o \) whenever \( F_0 \subseteq F_{A_0} \) and \( F_0 \subseteq F_{sp}(F_A) \) and for the soft subset \( F_{A_2} \), \( F_{A_2} \subseteq p(F_{A_2})^o = F_{A_2} \) whenever \( F_{A_2} \subseteq F_{A_2} \) and \( F_{A_2} \subseteq F_{sp}(F_A) \). Therefore \( F_{A_0} \) and \( F_{A_2} \) are soft \( Pg \)-open sets but the soft subset \( F_{A_4} \) is not a soft \( Pg \)-open set.

4.3. Proposition. Every soft \( P \)-open set is a soft \( Pg \)-open set

Proof: Let \( F_0 \) soft \( P \)-open set, then for every soft \( P \)-open set \( F_0, F_0 \subseteq p(F_0)^o \), we have every soft \( P \)-open set is soft \( Pg \)-open set. But the converse is not true in general.

4.4. Remark. A soft \( Pg \)-open set need not be soft \( P \)-open set. The following example supports our claim.

4.5. Example. Let as consider the soft subsets in Example 2.5. Let \((F_A, \bar{\tau})\) be a soft topological space in Example 4.2 where \( U = \{h_1, h_2\} \), \( A = \{e_1, e_2\} \), \( F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\} \) and \( \bar{\tau} = \{F_0, F_A, F_{A_4}, F_{A_6}\} \). \( G_{sp}(F_E) = \{F_0, F_A, F_{A_4}, F_{A_6}, F_{A_8}, F_{A_{12}}, F_{A_{14}}, F_{A_{16}}\} \). The soft set \( F_{A_4} \) is soft \( Pg \)-open set but it is not a soft \( P \)-open set.

4.6. Proposition
(i)If \( F_A \) and \( F_B \) are soft \( Pg \)-open sets then so is \( F_A \cup F_B \).
(ii)The soft union of two soft \( Pg \)-open sets is generally not a soft \( Pg \)-open set.

4.7. Proposition
\[ G_{sp}(F_E, \bar{\tau}) \subseteq G_{spg}(F_E, \bar{\tau}). \]

Proof: Let \( F_A \) be any soft \( P \)-open set. Then, \( F_A^c \) is soft \( P \)-closed and hence soft \( Pg \)-closed. This implies that \( F_A \) is soft \( Pg \)-open. Hence \( G_{sp}(F_E, \bar{\tau}) \subseteq G_{spg}(F_E, \bar{\tau}) \).
4.8. Definition. Let \((F_E, \tilde{r})\) be a soft topological space and \(F_A \subseteq F_E\). Then the pre generalized interior (soft Pg-interior) of \(F_A\) denoted by \(pg(F_A)^o\) is defined as the soft union of all soft pre generalized open subsets of \(F_A\).

4.9. Remark. Since arbitrary soft union of soft pre generalized-open sets is soft pre generalized open set, \(pg(F_A)^o\) is a soft pre generalized open set. Note that \(pg(F_A)^o\) is the biggest soft pre generalized open set that contained in \(F_A\).

4.10. Theorem. Let \((F_E, \tilde{r})\) be a soft topological space and \(F_A, F_B\) be a soft sets over \(U\). Then

(a) \(pg(F_A)^o \subseteq F_A\).

(b) \(F_A\) is soft pre generalized open iff \(F_A = pg(F_A)^o\).

(c) \(F_A \subseteq F_B\), then \(pg(F_A)^o \subseteq pg(F_B)^o\).

(d) \(pg(F_B)^o = F_B\) and \(pg(F_E)^o = F_E\).

(e) \(pg(F_A \cap F_B)^o = pg(F_A)^o \cap pg(F_B)^o\).

(f) \(pg(F_A \cup F_B)^o \subseteq pg(F_A)^o \cup pg(F_B)^o\).

(g) \(pg(pg(F_A)^o)^o = pg(F_A)^o\).

(h) (i) \((pg(F_A))^o \subseteq pg(F_A)^o\)

(ii) \((pg(F_A)^o)^o = pg(F_A)^o\)


4.11. Theorem. A soft set \(F_A\) is soft Pg-open in \((F_E, \tilde{r})\) and \(p(F_A)^o \subseteq F_O \subseteq F_A\), then \(F_O\) is soft Pg-open set.

Proof: Suppose that \(F_A\) is soft pre generalized open in \((F_E, \tilde{r})\) and \(p(F_A)^o \subseteq F_O \subseteq F_A\). Let \(F_B \subseteq F_O\) and \(F_B\) is soft Pg-closed set. Since \(F_O \subseteq F_B\) and \(F_B \subseteq F_O\), we have \(F_B \subseteq F_A\). Hence \(F_B \subseteq p(F_A)^o\). (Since \(F_A\) is soft pre generalized open). Since \(p(F_A)^o \subseteq F_O\), we have \(F_B \subseteq p(F_A)^o \subseteq p(F_O)^o\). Therefore, \(F_O\) is soft pre generalized open.

4.12. Note. Soft Pg-open set implies soft P-open set if it is a soft PT\(_{1/2}\) space.

5. Soft PT\(_{1/2}\) Spaces

In this section, we introduce the new soft lower separation axioms, namely soft pre T\(_{1/2}\)-space in soft topological space with the help of soft Pg- generalized closed sets.

5.1. Definition. A soft topological space \((F_E, \tilde{r})\) is said to be a soft pre T\(_{1/2}\) space (PT\(_{1/2}\) -space) if every soft Pg-closed set is soft P-closed set. The spaces where the class of soft P-closed sets and the soft Pg-closed sets coincide.

5.2. Example. Any soft discrete topological space \((F_E, \tilde{r})\) is soft PT\(_{1/2}\) -space. Since every soft subset of \(F_E\) is a soft P-closed sets in a soft discrete topology. Hence every soft subset of \(F_E\) is a soft Pg-closed set in a soft discrete topological space.

5.3. Example. Consider a soft topological space \((F_A, \tilde{r})\) in Example 2.5. Where \(U = \{h_1, h_2\}\), \(A = \{(e_1, e_2)\}, F_A = \{(e_1, \{h_1, h_2\})\}, (e_2, \{h_1, h_2\})\) and \(\tilde{r} = \{F_0, F_A, A_{A_1}, A_{A_2}\}\). The soft sets are defined as follows \(F_A = \{(e_1, \{U\})\}, F_{A_{10}} = \{(e_1, \{U\})\}, A_{HP} = \{F_0, F_A, A_{A_2}, F_{A_2}, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{13}}, F_{A_{14}}, F_{A_{15}}, F_{A_{16}}\}\). Therefore \((F_A, \tilde{r})\) is a soft PT\(_{1/2}\) -space over \(U\).

5.4. Remark

Every soft PT\(_{1/2}\) -space is a soft topological space.

5.5. Remark

A soft topological space need not be a soft PT\(_{1/2}\) -space.
5.6. Example

Let as consider the soft subsets in Example 2.5. Let \((F_A, \tilde{t})\) be a soft topological space where \(U = \{h_1, h_2\}, A = \{e_1, e_2\}, F_A = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\} and \(\tilde{t} = \{F_0, F_A, F_A, F_A, F_A\}, \tilde{t}^e = \{F_0, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A\}\). Hence, we have \(G_{sp}(F_E) = \{F_0, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A\}\), then \(F_{sp}(F_E) = \{F_0, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A, F_A\}\). The soft topological space \((F_A, \tilde{t})\) is not a soft PT\(_{1/2}\) space, because the soft set \(F_A\) is not a soft \(P\)-closed set.

5.7. Theorem. A soft topological space \((F_E, \tilde{t})\) is a soft PT\(_{1/2}\)-space if and only if \(G_{sp}(F_E) = G_{spg}(F_E)\) holds.

**Proof:** Necessity. Since the soft \(P\)-closed sets and the soft \(Pg\)-closed sets coincide by the assumption, \(G_{sp}(F_E) = G_{spg}(F_E)\) holds for every soft subset \(F_C\) of \((F_E, \tilde{t})\). Therefore, we have that \(G_{sp}(F_E) = G_{spg}(F_E)\).

Sufficiency. If \(G_{sp}(F_E) = G_{spg}(F_E)\), then \(F_E\) be a soft \(Pg\)-closed set. Therefore \(F_{sp}(F_E) = pG(\bar{F}_C)\) and hence \(F_C \subseteq G_{sp}(F_E)\). Thus \(F_C\) is soft \(P\)-closed set. Therefore \((F_E, \tilde{t})\) is soft PT\(_{1/2}\)-space.

5.8. Theorem. A soft topological space \((F_E, \tilde{t})\) over \(U\) is a soft PT\(_{1/2}\)-space if and only if, for each \(x \in U\), the soft singleton \(x_E\) is either soft \(P\)-open or soft \(P\)-closed.

**Proof:** Necessity. Suppose that for some \(x \in U\), \(x_E\) is not soft \(P\)-closed. Since \(F_E\) is the only soft \(P\)-open set containing \(x_E\), the soft set \(x_E\) is soft \(P\)-closed and so it is soft \(P\)-closed in the soft PT\(_{1/2}\)-space \((F_E, \tilde{t})\). Therefore \(x_E\) is soft \(P\)-open set.

Sufficiency. If \(G_{sp}(F_E) \subseteq G_{spg}(F_E)\), by Theorem 5.7, it is enough to prove that \(G_{spg}(F_E) \subseteq G_{sp}(F_E)\). Let \(F_0 \subseteq G_{spg}(F_E)\). Suppose that \(F_0 \not\subseteq G_{sp}(F_E)\). Then \(pG(\bar{F}_C) = F_0\) and \(p\bar{F}_C = F_0\) hold. There exists \(x \in U\) such that \(x \notin p\bar{F}_C\) and \(x \notin F_0\). Since \(x \notin p(F_0)\), there exist a soft \(Pg\)-closed set \(F_C\) such that \(x \notin F_C\) and \(F_C \supseteq F_0\). By the hypothesis, the soft singleton \(x_E\) is soft \(P\)-open or soft \(P\)-closed set.

Case (i)

The soft singleton \(x_E\) is soft \(P\)-open set: Since \(x_E\) is a soft \(P\)-open set with \(F_0 \subseteq x_E\), we have \(\bar{F}_C \subseteq x_E\). That is \(x \notin p(F_0)\). This contradicts the fact that \(x \notin p(F_0)\). Therefore \(F_0 \subseteq G_{sp}(F_E)\).

Case (ii)

If the soft singleton \(x_E\) is soft \(P\)-closed: Since \(x_E\) is a soft \(P\)-closed set containing the soft \(Pg\)-closed set \(F_C\), we have \(x_E \subseteq p(F_C) \supseteq p(F_0)\). Therefore \(x \notin p(F_0)\). This is a contradiction. Therefore \(F_0 \not\subseteq G_{sp}(F_E)\). Hence in both cases, we have \(F_0 \subseteq G_{sp}(F_E)\), that is \(G_{spg}(F_E) \subseteq G_{sp}(F_E)\).

5.9. Theorem. If \((F_E, \tilde{t})\) is a soft topological space, then the following statements are equivalent

(i) \((F_E, \tilde{t})\) is a soft PT\(_{1/2}\)-space.

(ii) Every soft subset of \(U\) is the soft intersection of soft \(P\)-open and soft \(P\)-closed set containing it.
Proof: (i)→(ii)  
If $F_E$ is a soft $PT_{1/2}$-space with $F_A \subseteq F_E$, then $F_A = \bigcap \{ F_E \setminus x_E : x \in F_A \}$ is the intersection of soft P-open and soft P-closed set containing it.

(ii)→(i)  
For each $x \in U, F_E \setminus x_E$ is the soft intersection of all soft P-open and soft P-closed sets containing it. Hence $F_E \setminus x_E$ is either soft P-open or soft P-closed. Therefore by Theorem 5.8, $(F_E, \mathcal{T})$ is a soft $PT_{1/2}$ -space.

5.10. Proposition. The soft subspace of a soft $PT_{1/2}$-space is soft $PT_{1/2}$-space.

Proof: Let $V$ is a soft subspace of a soft pre-$T_{1/2}$ space over $U$ and $x \in V \subseteq U$. Then $x_E$ is soft p-open or soft pre closed in $(F_E, \mathcal{T})$. (by Theorem 5.8) Therefore $x_E$ is either soft P-open or soft P-closed set in $V$. Hence $V$ is a soft $PT_{1/2}$ -space.

Conclusion: The initiation of notation of soft topological space was introduced by D.Molodtsov [5] in 1999. Many Mathematicians turned their attention to the various concepts of soft topological space. By this way, [6] Muhammad Shabir and Munazza Naz introduced the concept soft topological spaces. In 2013 J.Subhashini and C.Sekar [11] introduced the concept of soft pre-open sets. In this paper, we continue this work and introduce soft pre generalised -closed sets taking help of the soft pre open sets. Also we introduce soft $PT_{1/2}$ -Space. We discuss the relation between soft pre generalised closed sets and soft $PT_{1/2}$ spaces. Finally we prove every soft topological space need not be a soft $PT_{1/2}$ – Spaces.

Reference: