Performance Analysis of Conjugate Gradient and Recursive Least Square Adaptive Filters On Smart Antenna Systems

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Abstract: In this paper, Adaptive beam former design using conjugate gradient method (CGM) and RLS algorithm has been proposed. The performance of CGM has fast convergence rate than steepest descent [2], [12] and that it also has lower computational complexity when compared with the classic recursive least squares (RLS) algorithm. Simulation results reveal that RLS and CGM algorithms have high resolution for beam formation. However CGM has good performance to minimize MSE and better convergence as compared to RLS. Therefore, CGM is found more efficient algorithm to implement in the mobile communication environment to minimize MSE.

Keywords—Adaptive filtering algorithms, conjugate gradient method and RLS.

I.INTRODUCTION

Smart antenna can be used to achieve different benefits. By providing higher network capacity, it increases revenues of network operators and gives customers less probability of blocked or dropped calls. Adaptive Beam forming [1] is a technique in which an array of antennas is exploited to achieve maximum reception in a specified direction by estimating the signal arrival from a desired direction (in the presence of noise) while signals of the same frequency from other directions are rejected.

In many adaptive filtering algorithms based on the conjugate gradient (CG) method of optimization have been reported [3], [4]. In these works, several modifications have been proposed to improve the performance of the CG algorithm for various applications, but usually, the analysis of the proposed algorithms has not been shown. It is well known that the CG algorithm has a faster convergence rate than steepest descent [2] and that it also has lower computational complexity when compared with the classic recursive least squares (RLS) algorithm [3], but mostly, its analysis can only be found in the optimization and matrix computation literature. Here, we will describe, from the signal processing point of view, two of the CG algorithm implementations and analyze their performance in steady state. Some related implementation ideas can also be found in [3] and [7]. In addition, their performance under finite word-length effects will be discussed. Due to the highly nonlinear nature of the algorithms, a linear quantization model is used in the analysis of the LMS [6], NLMS [8] and RLS [5] algorithms, in general, cannot be applied.

In this paper is organized as follows. A brief review on adaptive filters is given in section II, overview on RLS technique in section III, and CGM technique in section IV. The simulation results are presented in Section V. Concluding remarks are made in Section VI.

II. ADAPTIVE FILTER

The principle of an adaptive filter is its time-varying, self-adjusting characteristics. An adaptive filter usually takes on the form of an FIR filter structure, with an adaptive algorithm that continually updates the filter coefficients, such that an error signal is minimized according to some criterion. The error signal is derived in some way from the signal flow diagram of the application, so that it is a measure of how close the filter is to the optimum. Most adaptive algorithms can be regarded as approximations to the Wiener filter, which is therefore central to the understanding of adaptive filters.
\[ y[n] = \sum_{k=0}^{N-1} c[k]x[n-k] \]

Here, the \( c[k] \) are time dependent filter coefficients (we use the complex conjugated coefficients \( c[k] \) so that the derivation of the adoption algorithm is valid for complex signals, too).

The block diagram of an adaptive filter is as shown in fig.1.

Fig.1. Block diagram of adaptive filter

III. RLS ADAPTIVE FILTER

The convergence speed of the LMS algorithm depends on the Eigen values of the array correlation matrix. The RLS algorithm does not require any matrix inversion computations as the inverse correlation matrix is computed directly. It requires reference signal and correlation matrix information. An important feature of the recursive least square algorithm is that its rate convergence is typically an order of magnitude faster than that of the simple least square.

Given an adaptive filter with an input \( x(n) \), an impulse response \( w(n) \) and an output \( y(n) \) you will get a mathematical relation for the transfer function of the system

\[ y(n) = w^T(n)x(n) \]

and

\[ x(n) = [x(n), x(n-1), x(n-2)... x(n-(N-1))] \]

\[ w^T(n) = [w_0(n), w_1(n), w_2(n) ... w_{N-1}(n)]^T \]

\( w_i^T(n) \) are the time domain coefficients

where \( H \) denotes the Hermitian (complex conjugate) transpose. The weight vector \( W \) is a complex vector. The process of weighting these complex weights \( w_1...w_{N-1} \) adjusted their amplitudes and phases such that when added together forms the desired beam.

In recursive least-square (RLS) algorithm [6], the weights are updated by the following equation

\[ W(n-1) = W(n-2) + K(n)[d(n) - W(n-2)X(n)] \]

The RLS algorithm does not require any matrix inversion computations as the inverse correlation matrix is computed directly. It requires reference signal and correlation matrix information.

IV. CONJUGATE GRADIENT METHOD

The problem with the steepest descent method has been the sensitivity of the convergence rates to the Eigen value spread of the correlation matrix. Greater spreads result in slower convergences. The convergence rate can be accelerated by use of the conjugate gradient method (CGM) [5]. The goal of CGM is to iteratively search for the optimum solution by choosing conjugate (perpendicular) paths for each new iteration. The method of CGM produces orthogonal search directions resulting in the fastest convergence. The path taken at iteration \( n + 1 \) is perpendicular to the path taken at the previous iteration \( n \). CGM is an iterative method, whose goal is to minimize the quadratic cost function. The algorithm of CGM as shown below

\[ K=0; xo=0; \]

\[ \text{While}([|r_k|^2] > \text{tolerance}) \text{ and } (k < \text{max_iter}) \]

\[ K++ \]

\[ If \ k = 1 \]

\[ P1=r0 \]

\[ Else \]

\[ B1=(r_{k-1}r_{k-2})/(p_{k-2}p_{k-2}) \]

\[ P_k=r_{k-1}+\beta_kp_{k-1} \]

\[ Endif \]

\[ S_k=Ap_k \]

\[ A_k=(r_{k-1}r_{k-2})/(p_k-s_k) \]

\[ R_k=r_{k-1}-\alpha_kS_k \]
The main disadvantage of normalized RLS is its high sensitivity to measurement noise. The normalized RLS algorithm lead to amplification of the measurement noise in low order filters especially when the reference signal power is low.

V. EXPERIMENTAL RESULTS

The performance of the algorithm is evaluated through radiation pattern and convergence analysis which are particularly attractive measurement of the wireless communications. In this paper we examined the signal model corresponding to uniform linear equally spaced array. When modeling an antenna array, we make the following assumptions

1) The spacing between array elements is $\lambda/2$.
2) There is no mutual coupling between elements.
3) All incidents fields can be decomposed into a discrete number of plane waves. That is there are finite numbers of signals.
4) The bandwidth of the signal incident on the arrays is small compared with the carrier frequency.

CGM algorithm places adaptively the maxima in the direction of desired user and nulls at the AOA of the interferer for various values of $N$. Simulation results prove that higher Value of antenna element gives better results.

![Fig.2. Rectangular Beampttern for CGM Algorithm with AOA desired user at 60° and Interference at 0°](image)

![Fig.3. Error signal for n=5, 6, 7 elements using CGM algorithm](image)

![Fig.4. Rectangular Beampttern for CGM Algorithm with AOA desired user at 60° and Interference at 0°](image)

The array factor plots for a spacing between the elements of quarter wave length and one eight wave length respectively. From these simulations it is evident that the optimum spacing beam between the elements is half wave length.
Fig.5. Rectangular Beam pattern for RLS Algorithm with AOA desired user at 60° and Interference at -20 for N=5, 6, 7 element

RLS algorithm places adaptively the maxima in the direction of desired user and nulls at the AOA of the interferer for various values of N. Simulation results prove that higher Value of antenna element gives better results.

Fig.6. Error signal for n=5,6,7 elements using RLS algorithm

Fig.7. Rectangular Beam pattern for RLS Algorithm with AOA desired user at 40° and Interference at 25°

The array factor plots for a spacing between the elements of quarter wave length and one eight wave length respectively. From these simulations it is evident that the optimum spacing beam between the elements is half wave length.

VI. CONCLUSION

In this paper, the non-blind adaptive beam forming algorithms such as RLS and CGM have been analyzed on a smart antenna system. It was noticed that increasing the number of elements of the antenna array ensures better performance. Conventionally, the LMS adaptive algorithm has been used to update the combining weights of adaptive antenna array. In an environment yielding an array correlation matrix with large Eigen values spread the algorithm converges with a slow speed. This problem is solved with the RLS and CGM algorithm by replacing the gradient step size with a gain matrix. It was noticed that increasing the number of elements of the antenna array ensures better performance. Also conclude that the optimum spacing beam between the elements is half wave length.

REFERENCES


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