Review: Sparse Representation for Face Recognition Application

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Abstract— In recent years, signal processing has come under pressure to accommodate the increasingly high-dimensional data generated by modern sensing systems. In many cases these high-dimensional signals contain relatively little information compared to their ambient dimensionality. Thus signals can often be well-approximated as a sparse linear combination of elements from a known basis or dictionary. Sparse models are exploited only after acquisition, typically for compression.

In this paper we discuss how face detection problem is solved using sparse representation with the touch of compressive sensing theory. We consider sparsity based classification here.

Keywords— Face Recognition, Sparse Representation, l_1-minimization

I. INTRODUCTION

Although we can find many other identification and verification techniques, such as password, PIN token systems, the face recognition has its own advantages as it is considered as a passive system to verify and identify people. Thus within the last several decades, face recognition has received extensive attention due to its wide range of application from identity authentication, security system to human-computer interaction. Also it is one of the most important research topics in computer vision and pattern recognition. An identification technique is closer to the way human being recognize each other. Although human perception is very effective in identifying human subjects from facial images, most contemporary face recognition systems have failed to achieve equally good recognition accuracy and there can be numerous factors affecting their performance. A lot of methods have been proposed for face recognition [1,2,3]. These methods are either i.e. geometric feature based methods or template based methods. These methods use the raw face images for face recognition task. They pay less attention to the face representation.

In this paper, we discuss about a recent and efficient solution to robust face recognition [4,5] which has been motivated by the emerging theory of compressive sensing (CS). This CS method reformulates the face recognition problem as a sparse representation problem. Compressive sensing (or compressed sensing) is an emerging research area. It provides a new way to acquire and represent sparse signals. The original goal of this work was not mere classification but rather representation and compression of signals using lower sampling rates than the Shannon- Nyquist bound [8]. The work in this paper is guided by [6, 7, 8].

The basic idea behind this framework is to consider the new input image as a sparse linear combination of training images. In this method we seek the sparsest linear representation of the sample image with respect to a known dictionary or database to which the particular query sample image belongs. Here we are using Yale B database for 30 classes. Each class contains 60 different images of size 192 x 168, of one person. We are using half database as a training set and half for testing purpose.

The complete face recognition using sparse representation is explained in 4 steps of Sparse Representation based Classification (SRC) algorithm [6].

Step1: Test samples as a sparse linear combination of Training samples
(I) Generation of A matrix: We define a new matrix A for the entire training set as the collection of the n training sample of all k classes where each class contain all the available images of kth person.

\[ A = [A_1, A_2, A_3, \ldots, A_k] \]  

(1)

Considering total m pixel in one gray scale image, each image will form one column \( v \) with dimension m X 1. Then we arrange the available ni training samples from each \( i \)-th class as columns of a matrix \( A_i \) where

\[ A_i = [v_{i,1}, v_{i,2}, \ldots, v_{i,n}] \]  

(2)

(II) Test samples as sparse linear combination of training samples:

The linear representation of any test image \( y \) can be written in terms of all training samples as

\[ y = A x_0 \epsilon \mathbb{IR}^{m} \]  

\[ [y]_{m 	imes 1} = [A]_{m 	imes n} [x]_{n 	imes 1} \]  

(3)

For higher accuracy the solution of above equation should be a column matrix with very less no of non zero entries.

(III) Sparsity: The signal \( x \) is K-sparse when it has at most K nonzero. Sparse signals are well approximated as a linear combination of just a few elements from a known basis or dictionary. Sparsity lies at the heart of the emerging field of compressive signal processing (CSP). In particular the sparse models are the Solutions to Curse of dimensionality. The multiscale wavelet transform provides nearly sparse representations for natural images.

Thus as a solution of (3) we should achieve \( x_0 \) as sparse as possible and our aim to seek sparsest solution for our recognition problem, where

\[ x_0 = [0, \ldots, 0, a_{1}, a_{2}, \ldots, a_{n}, 0, \ldots, 0]^T \epsilon \mathbb{IR}^{m} \]

is sparse in nature and it is a coefficient vector which contain only those non zero coefficient values which are associated with \( i \)-th class, remaining entries are zero.

**Step 2: Normalizing Sensing matrix A:**

The images available as training samples in the database contribute in the generation of columns of sensing matrix. The sparse linear combination of this collection of training sample will be the solution to our classification problem. Thus we have to normalize each column of matrix A using l2 norm and we have to achieve unit l1-norm of each column(image) .Euclidean or l2-norm is given by:

\[ \|x\|_2 = \min x \quad\text{subject to } Ax = y \]

**Step 3: Transformation from image space to feature space and need of dimension reduction:**

In overdetermined System, the number of equations is greater than the number of variables. Thus, there are no solutions, usually. The solutions for such systems are given by Least Square Methods, where \( x \) is found so as to minimize \( \|Ax-Y\|^2 \).

**Overdetermined System**

In underdetermined system, number of equations are less than number of unknowns. It has many many solutions. Therefore select minimum l1-Norm solution. i.e. \( \|x\|_1 \) should be minimum. Most of the time, \( \|x\|_1 \) norm solution is a sparsest solution. Sparsest solution will have the least number of nonzero entries. It is very hard to solve such problems, thus Linear Programming are used to solve it. In our work we achieve sparsest solution by solving the linear system of equations \( y = Ax \).

Since each image available in database is with resolution 192 X 168 pixels, the dimension value \( m \) reaches nearby 10^5 pixels. Thus though we are handling linear programming for \( y = Ax \) system, in such high dimensionality case, it becomes tough to obtain solutions of such systems. As a solution to this trouble, before going for classification problem, all the gray scale images are processed for dimensionality reduction[6]. It is also referred as image space to feature space transformation. This transformation considers feature space dimension \( d \ll m \). Thus system

\[ [y]_{m \times 1} = [A]_{m \times n} [x]_{n \times 1} \]

becomes

\[ [y]_{d \times 1} = [A]_{d \times n} [x]_{n \times 1} \]

Here chosen dimension \( d \) is much less than total training sample images \( n \). In such case

\[ \|y\| = \|R\| \|A\| \|x\| = \|I\| \|R\|^d \]

becomes underdetermined system. Solving such systems also as l1 minimization problem , we get desired sparse solution as \( x_0 \).

In our sparse representation approach to recognition, for dimensionality reduction, here we have first downsampled all the images in database thus instead of 10^5 pixels of image with resolution 192 X 168, now we have only 120 pixels of each downsampled image with resolution 12 X 10. Matrix A has dimension 120 X n and system is underdetermined. Downsampling can also be replaced by wavelet coefficients or Eigen faces of each image, which can also provide sparse solution to a underdetermined system.
(IV) Minimum $l_1$ norm solution: As we have underdetermined system $y=Ax$, it can not have the unique solution. In such cases, solution by minimum $l_2$ norm can be considered.

$$l_2: \ x_2 = \text{arg min} \|x\|_2 \quad \text{subject to} \ Ax=y$$

But minimum $l_2$ norm solution is not efficient for recognition of test sample $y$. Ideally test sample $y$ should get represented using the training samples from the same class only. If in our system number of classes $k$ is relatively large, then solution to this system or matrix $x_0$ should be sparse in nature. But $l_2$ norm solution - $x_2$ is dense in nature with large nonzero entries corresponding to training samples from many different classes and it doesn’t lead to sparsity which is not desirable.

**Step 4: To find sparse solution via $l_1$-Minimization:**

$l_1$- minimization solves the minimum 1-norm solution to an underdetermined linear system $y=Ax$. Theory of compressive sensing has proved that, under certain conditions 11 minimization solution is the sparsest solution to that system.

Specifically for our approach, for unknown solution $x_0 \in \mathbb{R}^d$, vector $y \in \mathbb{R}^d$ can be generated by linear projection $y=Ax$. Assuming sensing matrix to be full rank i.e. $d<n$ then $l_1$ min program solves above system as:

$$(P1): \ \text{min} \|x\|_1 \quad \text{subject to} \ y=Ax$$

As number of entries in $y$ is smaller than the number of unknowns in $x$, the formulation of $(P1)$ constitutes a linear inverse problem, and we achieve solution $x$ in its sparsest form. This sparsity seeking property of solution by 11-min has tremendous applications in data compression, image processing, sensor network and computer vision [6,8,9].

Traditional formulation of $l_1$-min as a linear programming (LP) problem as in Basis Pursuit (BP) has high computational complexity. Alternatively greedy algorithms like orthogonal matching pursuit (OMP) and least angle regression, are significantly faster than using LP. These greedy algorithm find sufficiently good solutions to approximate $(P1)$[10].

**III. EXPERIMENT**

In this section we discuss the performance for the SRC algorithm for face recognition which is capable of classify the query image in its belonging class and identify it with high accuracy.

In our work we are using Extended Yale database containing 38 different persons with 65 RGB images of each, from where we have selected the subset of 30 persons with 60 images of each person in different varying illuminations are selected. Then we have divided selected subset of database in two parts. One set of 30 persons with 30 images of each one is used as a database of training sample images. Another set of same 30 persons with different 30 images of each is kept for testing purpose. To verify the efficiency of given algorithm, Out of all 30 images in testing sample set,7 images of each person are taken in very less illumination (0 to 20%), 7 images are taken in very high (80% to 100%) illumination and remaining images are taken in moderate illumination (20% to 80%). After converting all the images into gray scale form we get each image with dimension 192 X 168.

Now as discussed in step 3 of (2), to reduce image dimensions we downsample all the images. Here all the images are downsampled from 192 X168 to 12 X 10 pixels. Following the first step in SRC algorithm, sensing matrix $A$ of dimension $120 \times 900$ is designed containing 900 images and its each column is generated by one image with 120 pixels. Then we normalized each column of matrix $A$ with unit $l_2$ normalization method. For testing purpose, we chose one random image which is clearly visible and with moderate illumination. This testing image will form our matrix $y$, with dimension $120 \times 1$ and the system $y=Ax$ becomes underdetermined. Applying 11 minimization algorithm to this linear system, the sparse nature of solution $x$ classifies the matrix $y$ in its accurate class i.e. the image gets recognised correctly. The better accuracy of the recognition process highly depends upon the high degree of sparsity of unknown matrix $x$. The correct recognition rate in this case, we got is approximately 90%. Later to verify the efficiency of the algorithm, we tested the images with very less illumination (0 to 20%) and the images with very high (80% to 100%) illumination. In both cases we got 50% accuracy in recognition. Next we considered the case, if the query image is not from the assigned database, then for such unknown query image, the optimum algorithm should detect it as unknown subject. When a image of any unknown person which is not from our database, is considered as a test image, we get result as unclassified with 80% accuracy and with 20% error rate. In such case Error or misclassification means algorithm classifies such unknown image in any other class and gives faulty recognition.

**IV. CONCLUSION**

Based on Sparse representation concept, we have discussed a more efficient method of classification in face recognition application. For compression purpose we have done the downsampling of each image, our next task will be the verification of the same classification method using the highest wavelet coefficients of the subject image. Further work will proceed with use of Eigen faces instead. Our future work will mainly focus on achieving the equally high accuracy in robustness to Occlusion or Corruption of images.

**REFERENCES**


