Performance Analysis of MIMO Systems Using OSTBCs

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Abstract – The use of multiple antennas at transmitter as well as at receiver of a communication system gaining ground in wireless network. This method is used for achieving spatial diversity to combat fading without expanding the bandwidth of the transmitted signals. This paper presents the analyses of the BER performance of the different modulation schemes using up to four transmit as well as receive antennas. In this analysis we have used orthogonal space – time block code diversity system over flat Rayleigh fading channels. Independent fading between diversity channels is assumed. Diversity consists of transmitting a single space-time coded stream through all antennas. Spatial multiplexing increases network capacity by splitting a high rate signal into multiple lower rate streams and transmitting them through the different antennas. In spatial multiplexing, the receiver can successfully decode each stream given that the received signals have sufficient spatial signatures and that the receiver has enough antennas to separate the streams. The result of using these MIMO techniques is higher data rate or longer transmit range without requiring additional bandwidth or transmit power. This paper presents a detailed study of diversity coding for MIMO systems. Different space-time block coding (STBC) schemes including Alamouti’s STBC for 2 transmit antennas as well as orthogonal STBC for 3 and 4 transmit antennas are explored. Finally, these STBC techniques are implemented in MATLAB and analyzed for performance according to their bit-error rates using BPSK, QPSK, 16-QAM, and 64-QAM modulation schemes.

Index Terms – Multiple input and multiple output (MIMO) systems, Orthogonal Space – time Block Coding (OSTBC), Rayleigh fading channels, and Bit Error Rate (BER).

I. Introduction

Wireless system designers are facing a number of challenges. These include the limited availability of the radio frequency spectrum and complex space – time varying wireless environment. In addition, there is an increasing demand for higher data rates, better quality of service, and higher network capacity. In recent years, Multiple – Input and Multiple – Output (MIMO) systems have emerged as a most promising technology in these measures. MIMO communications systems can be defined by considering that multiple antennas are used at the transmitting end as well as at the receiving end. The core idea behind MIMO is that signals sampled at the spatial domain at both ends are combined in such a way that they either create effective multiple parallel spatial data pipes, therefore, increasing the data rates, and/or add diversity to improve the quality (BER) of the communications.

OSTBC can achieve full diversity gain with low complexity maximum likelihood (ML) decoding by a linear processing receiver. OSTBC can transform multiple – input multiple – output (MIMO) fading channels into equivalent single input single output (SISO) additive white Gaussian noise (AWGN) channels for a fixed channel realization. It is known that, for the same diversity order, the error rate for OSTBC MIMO systems equivalent to that for maximal ratio combining (MRC) systems except with a performance loss. Information theory predicts that MIMO channels are able to provide huge gain in terms of reliability and transmission rate. It is important to note that each antenna element on a MIMO system operates on the same frequency and therefore does not require extra bandwidth. Also, for fair comparison, the total power through all antenna elements is less than or equal to that of a single antenna system, i.e.

\[ \sum_{n=1}^{N} p_n \leq P \]  

(1)

Where, N is the total number of antenna elements, \( p_n \) is the power allocated through the \( k \)th antenna element, and P is the power if the system had a single antenna element. Effectively, (1) ensures that a MIMO system consumes no extra power due to its multiple antenna elements.

As a consequence of their advantages, MIMO wireless systems have captured the attention of international standard organizations. The use of MIMO has been proposed multiple times for use in the high-speed packet data mode of third generation cellular systems (3G) as well as the fourth generation cellular systems (4G). MIMO has also influenced wireless local area networks (WLANs) as the IEEE 802.11n standard exploits the use of MIMO systems to acquire throughputs as high as 600Mbps.

This paper presents the analyses of the BER performance of the different STBC techniques including the system model, capacity analysis, and channel models. Focus is then given to spatial diversity, specifically to space time block codes (STBC). We discuss Alamouti’s STBC as well as other orthogonal STBC for 3 and 4 transmit antennas and finally show simulation results and analysis.

The paper is organized as follows. In Section II, important general background information on MIMO is provided. Next, different STBC techniques are explained in Section III. The simulation methodology is discussed in Section IV. Results and analysis are presented in Section V. Finally, Section VI concludes this paper.
II. Background of MIMO communication systems

When multiple input/multiple output (MIMO) systems were described in the mid-to-late 1990s by Gerard Foschini and others, the astonishing bandwidth efficiency of such techniques seemed to be in violation of the Shannon limit. But, there is no violation because the diversity and signal processing employed with MIMO transforms a point-to-point single channel into multiple parallel or matrix channels, hence in effect multiplying the capacity. MIMO offers higher data rates as well as spectral efficiency. So clear is this advantage that many standards have already incorporated in MIMO. ITU uses MIMO in the High Speed Downlink Packet Access (HSPDA), part of the UMTS standard. MIMO is also part of the 802.11n standard used by wireless router as well as 802.16 for Mobile WiMax. The LTE standard also incorporates MIMO.

What is MIMO as compared to a traditional communications channel? A traditional communications link, which we call a single-in-single-out (SISO) channel, has one transmitter and one receiver. But instead of a single transmitter and a single receiver we can use several of each. The SISO channel then becomes a multiple-in-multiple-out, or a MIMO channel; i.e. a channel that has multiple transmitters and multiple receivers. Traditional wireless systems are affected by multipath propagation. In MIMO systems, however, this multipath effect is exploited to benefit the user. In fact, the separability of parallel streams depend on the presence of rich multipath. The reason for this effect will become apparent as the System Model is described in Section II-A below.

A. System Model

MIMO systems are composed of three main elements, namely the transmitter (TX), the channel (H), and the receiver (RX). In this paper, Nt is denoted as the number of antenna elements at the transmitter, and Nr is denoted as the number of elements at the receiver. Figure 1 depicts such MIMO system block diagram. It is worth noting that system is described in terms of the channel. For example, the Multiple-Inputs are located at the output of the TX (the input to the channel), and similarly, the Multiple-Outputs are located at the input of the RX (the output of the channel).

The channel with Nr outputs and Nt inputs is denoted as a \( N_r \times N_t \) matrix, where each entry \( h_{ij} \) denotes the attenuation and phase shift (transfer function) between the \( j \)th transmitter and the \( i \)th receiver. It is assumed throughout this paper that the MIMO channel behaves in a “quasi-static” fashion, i.e. the channel varies randomly between burst to burst, but fixed within a transmission of single burst. This is a reasonable and commonly used assumption as used in an indoor channel where the time of change is constant and negligible compared to the time of a burst of data.

\[
\begin{bmatrix}
    h_{1,1} & h_{1,2} & \cdots & h_{1,N_t} \\
    h_{2,1} & h_{2,2} & \cdots & h_{2,N_t} \\
    \vdots     & \vdots     & \ddots & \vdots     \\
    h_{N_r,1} & h_{N_r,2} & \cdots & h_{N_r,N_t}
\end{bmatrix}
\]

Fig. 1 MIMO antenna configuration

\[
H = \begin{bmatrix}
    h_{1,1} & h_{1,2} & \cdots & h_{1,N_t} \\
    h_{2,1} & h_{2,2} & \cdots & h_{2,N_t} \\
    \vdots     & \vdots     & \ddots & \vdots     \\
    h_{N_r,1} & h_{N_r,2} & \cdots & h_{N_r,N_t}
\end{bmatrix}
\]

The MIMO signal model is described as

\[
y_1 = \sum_{n=1}^{N_t} h_{1,n} \cdot x_n + z_1 \\
y_2 = \sum_{n=1}^{N_t} h_{2,n} \cdot x_n + z_2 \\
\vdots \\
y_{N_r} = \sum_{n=1}^{N_t} h_{N_r,n} \cdot x_n + z_{N_r}
\]

\[
\rightarrow y = H x + Z
\]

where \( y \) is the received vector of size Nr x 1, \( H \) is the channel matrix of size Nr x Nt, \( x \) is the transmitted vector of size Nt x 1, and \( z \) is the noise vector of size Nr x 1. Each noise element is typically modelled as independent identically distributed (i.i.d.) white Gaussian noise with variance \( N_t/2 \cdot SNR \).

An explanation for this model is as follows. The transmitted signals are mixed in the channel since
they use the same carrier frequency. At the receiver side, the received signal is composed of a linear combination of each transmitted signal plus noise. The receiver can solve for the transmitted signals by treating eqn. (3) as a system of linear equations. If the channel $H$ is correlated, the system of linear equations will have more unknowns than equations. One reason correlation between signals can occur is due to the spacing between antennas. To prevent correlation due to the spacing, they are typically spaced at least $\lambda_{c}/2$, where $\lambda_{c}$ is the wavelength of the carrier frequency. The second reason correlation can occur is due to lack of multipath components. It is for this reason that rich multipath is desirable in MIMO systems. The multipath effect can be interpreted by each receive antenna being in a different channel. For this reason, the rank of a MIMO channel is defined as the number of independent equations offered. It is important to note that 
$$\text{rank}(H) \leq \min(N_{r},N_{t})$$

and therefore the maximum number of streams that a MIMO system can support is upper-bounded by $\min(N_{r},N_{t})$.

**B. Channel Models**

Channel models for MIMO systems can be either simple or very complex, depending on the environment modeled and the desired accuracy. There are two different techniques for modeling MIMO channels. One method is to calculate the MIMO channel matrix according to a physical representation of the environment. The channel matrix in such a physical model would depend on physical parameters such as the angle of arrival (AOA), angle of departure (AOD), and time of arrival (TOA) [16]. In [13], Molisch presents a physical MIMO model and provides typical physical parameters for both macro and microcell environments. As expected, these type of deterministic models are highly complex. Another technique to model MIMO channels, is to model the channel analytically. Such a model treats all channels between each transmit antenna to each receive antenna as SISO channels. This type of model assumes that the channels are independent and identically distributed (i.i.d.). However, depending on the environment modeled, this assumption is rarely true. The reason is that MIMO channels can experience spatial correlation between links [16]. It is possible to generate a MIMO channel with a specific correlation matrix. The channel correlation matrix is usually measured in the field and it is tied to the environment setup such as antenna element patterns, spacing between antennas, and surrounding reflectors. Since one of the main goals of this paper is to compare the performance of different STBC schemes, the channel model is chosen such that the correlation does not interfere with the performance as such. Next, two channel models are discussed for the case of non-line-of-sight (NLOS) and the case of line-of-sight (LOS) respectively.

1) **NLOS Environment**: A typical model used in research to model NLOS scenarios is the Rayleigh model. The Rayleigh model assumes NLOS, and is used for environments with a large number of scatterers. The Rayleigh model has independent identically distributed (i.i.d.) complex, zero mean, unit variance channel elements and is given by

$$h_{lp} = \frac{1}{\sqrt{2}} (\text{Normal}(0,1) + j\text{Normal}(0,1))$$

(5)

This model results in an approximation which improves as the spacing between antennas become large compared to the wavelength $\lambda$.

2) **LOS Model**: The MIMO channel matrix for the LOS scenario is given by:

$$H = \sqrt{\frac{k}{1+k}} H_{LOS} + \sqrt{\frac{1}{1+k}} H_{NLOS}$$

(6)

Where $k (dB) = 10\log_{10} \frac{P_{LOS}}{P_{NLOS}}$.

In (6), $H_{LOS}$ is a rank-one matrix corresponding to the LOS component, and the $H_{NLOS}$ corresponds to the NLOS components. In (7), $P_{LOS}$ is the power due to the LOS component, and $P_{NLOS}$ is due to the power of the NLOS component. The $H_{LOS}$ component is usually modeled as eqn. (5). In SISO systems, the higher the K factor, the smaller the fade margin needed. For MIMO systems, the higher the K factor, the more dominant the rank-one $H_{LOS}$ component will be, and consequently, the less dominant the $H_{NLOS}$ component will be. However, for the scenario of rich multipath, simulations and measurements have shown that the LOS component rarely dominates.

**III. MIMO system and SPACE-TIME BLOCK CODING**

A MIMO system with $N_{t}$ transmit antennas and $N_{r}$ receive antennas is shown in figure 1. From the figure, it seems that space time coding is charged to perform serial – to – parallel and parallel – to- serial conversion of the transmitted information data. The block responsible for serial to parallel transformation at the transmitter is the Space – Time Encoder. The opposite operation, i.e., the parallel to serial conversion, is performed in the Space – Time Decoder at the receiver side. However, Space – Time coding is more than these transformations, since these codes can realize at the same time several communication processes such as channel encoding / decoding modulation / demodulation, multiplexing / de-multiplexing or equalization. In order to have a MIMO system comparable to SISO system, the sum of the transmitted powers of all antennas $N_{t}$ must be equal to the transmitted by the SISO system and denoted by $P$. Hence, the transmitted power from each antenna is $P/N_{t}$.
Space – Time Block Codes (STBCs) are the simplest type of spatial temporal codes that exploit the diversity offered in system with several transmit antennas. In 1998, Alamouti designed a simple diversity technique for system having two transmit antennas [1]. This method provides full code rate and requires simple linear operations at both transmission and reception side. The encoding and decoding processes are performed with blocks of transmission symbols. Alamouti’s simple transmit diversity scheme was extended in [6] using theory of orthogonal design for larger number of antennas. These codes are referred in literature as Orthogonal Space – Time Block Codes (OSTBCs).

A. Alamouti Space-Time Code

A complex orthogonal space-time block code for two transmit antennas was developed by Alamouti. In the Alamouti encoder, two consecutive symbols x1 and x2 are encoded with the following space-time codeword matrix:

\[
X = \begin{bmatrix}
    x_1 & -x_2 \\
    x_2 & x_1
\end{bmatrix}
\]  

(8)

Alamouti encoded signal is transmitted from the two transmit antennas over two symbol periods. During the first symbol period, two symbols x1 and x2 are simultaneously transmitted from the two transmit antennas. During the 2nd symbol period, these symbols are transmitted again, where \(-x_2^*\) is transmitted from the first transmit antenna and \(x_1^*\) transmitted from the second transmit antenna [1]. In this paper, the columns of each coding scheme represents a different time instant, while the rows represent the transmitted symbol through each different antenna. Note that the Alamouti code word X in equation (8) is a complex orthogonal matrix, i.e.,

\[
XX^H = \begin{bmatrix}
    |x_1|^2 + |x_2|^2 & 0  \\
    0 & |x_1|^2 + |x_2|^2
\end{bmatrix}
\]

\[
= \begin{bmatrix}
    |x_1|^2 + |x_2|^2 & 1 \\
    1 & |x_1|^2 + |x_2|^2
\end{bmatrix}
\]

(9)

Where I_2 is the 2x2 identity matrix. We know that the code rate is defined as

\[
R = \frac{1}{T}
\]

(10)

Where, N = number of symbols transmitted by a codeword over T number of time slots [6].

In Alamouti code there are two symbols transmitted over two symbol periods and hence the code rate is 1. The diversity gain is 2 because same symbol is transmitted twice through two transmit antennas. Note that the diversity analysis is based on ML signal detection at the receiver side. We now discuss ML signal detection for Alamouti space-time coding scheme. Here, we assume that two channel gains, \(h_1(t)\) and \(h_2(t)\), are time-invariant over two consecutive symbol periods [8], that is

\[
h_1(t) = h_1(t + T_s) = h_1 = |h_1|e^{j\theta_1}
\]

\[
h_2(t) = h_2(t + T_s) = h_2 = |h_2|e^{j\theta_2}
\]

(11)

Where \(|h_1|\) and \(\theta_1\) denote the amplitude gain and phase rotation over the two symbol periods, \(i = 1, 2\).

From equation (8)

\[
X = \begin{bmatrix}
    x_1 & -x_2 \\
    x_2 & x_1
\end{bmatrix}
\]

(12)

\[
H = [h_1 \ h_2]\text{, from equation (3)}
\]

\[
y = Hx + z
\]

\[
z = \begin{bmatrix}
    z_1 \\
    z_2
\end{bmatrix}
\]

\[
\Rightarrow y = [h_1 \ h_2]\begin{bmatrix}
    x_1 & -x_2 \\
    x_2 & x_1
\end{bmatrix} + \begin{bmatrix}
    z_1 \\
    z_2
\end{bmatrix}
\]

\[
= \begin{bmatrix}
    h_1x_1 + h_2x_2 \\
    -h_1x_2^* + h_2x_1^*
\end{bmatrix} + \begin{bmatrix}
    z_1 \\
    z_2
\end{bmatrix}
\]

\[
\rightarrow y_1 = h_1x_1 + h_2x_2 + z_1
\]

\[
y_2 = -h_1x_2^* + h_2x_1^* + z_2
\]

(13)

Taking complex conjugation of the second received signal, i.e.,

\[
y_2^* = -h_1^*x_2 + h_2^*x_1 + z_2^*
\]

and we have the following matrix vector equation

\[
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} = \begin{bmatrix}
    h_1 & h_2 \\
    h_2^* & -h_1^*
\end{bmatrix}\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} + \begin{bmatrix}
    z_1 \\
    z_2
\end{bmatrix}
\]

(14)

In the course of time, from time t to t+T, the estimates for channels, \(\hat{h}_1\) and \(\hat{h}_2\) are provided by the channel estimator. In the following discussion, however, we assume an ideal situation in which the channel gains, \(h_1\) and \(h_2\), are exactly known to the receiver. Then the transmit symbols are now two unknown variables in the matrix of Equation (14). Multiplying both sides of Equation (14) by the Hermitian transpose of the channel matrix, that is

\[
\begin{bmatrix}
    h_1^* & h_2^* \\
    h_2^* & -h_1^*
\end{bmatrix}\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} = \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} + \begin{bmatrix}
    h_1^*z_1 + h_2^*z_2^* \\
    h_2^*z_1^* + -h_1^*z_2^*
\end{bmatrix}
\]

\[
= (|h_1|^2 + |h_2|^2)\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} + \begin{bmatrix}
    h_1^*z_1 + h_2^*z_2^* \\
    h_2^*z_1^* + -h_1^*z_2^*
\end{bmatrix}
\]

\[
\begin{bmatrix}
    \tilde{y}_1^* \\
    \tilde{y}_2^*
\end{bmatrix} = \begin{bmatrix}
    h_1^* & h_2^* \\
    h_2^* & -h_1^*
\end{bmatrix}\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix}
\]

(15)

In Equation (16), we note that other antenna interference does not exist anymore, that is, the unwanted symbol \(x_2\) dropped out of \(y_1\), while the unwanted symbol \(x_1\) dropped out of \(y_2\). This is attributed to complex orthogonality of the Alamouti code in Equation (12). This particular feature allows for simplification of the ML receiver structure as follows:

\[
x_{i,\text{ML}} = Q\left(\tilde{y}_i^* (|h_1|^2 + |h_2|^2)\right), i = 1, 2.
\]

(17)
where \( Q(\cdot) \) denotes a slicing function that determines a transmit symbol for the given constellation set. The above equation implies that \( x_1 \) and \( x_2 \) can be decided separately, which reduces the decoding complexity of original ML-decoding algorithm from \( |C|^2 \) to \( 2|C|^2 \) where \( C \) represents a constellation for the modulation symbols, \( x_1 \) and \( x_2 \). Furthermore, the scaling factor \( (|h_1|^2 + |h_2|^2) \) in Equation (16) warrants the second-order spatial diversity, which is one of the main features of the Alamouti code.

### B. Generalization of Space-Time Block Coding

In the previous section, we have shown that thanks to the orthogonality of the Alamouti spacetime code for two transmit antenna cases, ML decoding at the receiver can be implemented by simple linear processing. This idea was generalized for an arbitrary number of transmit antennas using the general orthogonal design method in [2][Tarokh & Jafarkhani 1999]. Two main objectives of orthogonal spacetime code design are to achieve the diversity order of \( N_T N_R \) and to implement computationally efficient per-symbol detection at the receiver that achieves the ML performance.

**The general structure of space-time block encoder.**

The output of the spacetime block encoder is a codeword matrix \( X \) with dimension of \( N_t \times T \), where \( N_t \) is the number of transmit antennae and \( T \) is the number of symbols for each block. Let a row vector \( x_i \) denote the \( i \)-th row of the codeword matrix \( X \) (i.e. \( X_i = x_i^1, x_i^2, ..., x_i^T \) \( i = 1, 2, ..., N_t \)). Then, \( x_i \) will be transmitted by the \( i \)-th transmit antenna over the period of \( T \) symbols. In order to facilitate computationally-efficient ML detection at the receiver, the following property is required: \( XX^H = C( |x_1|^2 + |x_2|^2 + ... + |x_T|^2 ) \) \( = C |X_i|^2 I_{N_t} \) \( (19) \)

where \( C \) is a constant. The above property implies that the row vectors of the codeword matrix are orthogonal to each other, that is,

\[
x_i x_j^H = \sum_{t=1}^{T} x_i^t (x_j^t)^* = 0, \quad i \neq j, \quad i, j \in \{1, 2, ..., N_t\} \quad (20)
\]

### C. Orthogonal Space-Time Block Codes

The Alamouti scheme discussed in Section III-A is part of a general class of STBCs known as Orthogonal Space-Time Block Codes (OSTBCs)[1]. The authors of [7] apply the mathematical framework of orthogonal designs to construct both real and complex orthogonal codes that achieve full diversity. For the case of real orthogonal codes, it has been shown that a full rate code can be constructed. However, for \( N_T > 3 \), it has been known that there does not exist a complex space-time code that satisfies two design goals of achieving the maximum diversity gain as well as the maximum coding rate at the same time. Consider the following examples for \( N_T = 3 \) and \( N_T = 4 \). Complex modulation techniques are of interest in this paper and therefore real orthogonal codes are not discussed. In next sections, the full diversity complex orthogonal codes presented in for different rates are briefly introduced [7].

1) Orthogonal Space-Time Block Codes for \( N_T = 3 \): For the case of 3 transmit antennas, Tarokh et al. construct block codes with 1/2 and 3/4 coding rate and full diversity 3Nr.

a) \( N_T = 3 \) with Rate 1/2: The full diversity, rate 1/2 code for \( N_T = 3 \) is given by [7]:

\[
X_{\text{3, complex}}^{1/2} = \begin{bmatrix}
x_1 - x_2 - x_3 - x_4 & x_2^* - x_3^* - x_4^* - x_1^*
-x_2 - x_3 - x_4 & x_1^* - x_2^* - x_3^* - x_4^*
-x_3 - x_4 & x_1^* - x_2^* - x_3^* - x_4^*
\end{bmatrix}
\]

(21)

\[
X_{\text{3, complex}}^{3/4} = \begin{bmatrix}
x_1 & x_2 & x_3
-x_2 & x_1 & x_3
-x_3 & x_2 & x_1
\end{bmatrix}
\begin{bmatrix}
\frac{x_1}{\sqrt{2}} & \frac{x_2}{\sqrt{2}} & \frac{x_3}{\sqrt{2}}
\frac{x_2}{\sqrt{2}} & \frac{x_1}{\sqrt{2}} & \frac{x_3}{\sqrt{2}}
\frac{x_3}{\sqrt{2}} & \frac{x_2}{\sqrt{2}} & \frac{x_1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix}
M_1
M_2
\end{bmatrix}
\]

(22)
Where, 
\[ M_1 = (-x_1 - x_1^* + x_2 - x_2^*)/2 \quad \text{and} \]
\[ M_2 = (x_2 + x_2^* + x_1 - x_1^*)/2 \]
b) \( N = 4 \), with Rate 1/2 & 3/4: full diversity, is given by
\[
\begin{align*}
X^1_{\text{complex}} &= \begin{bmatrix}
-x_1 - x_2 & -x_3 & x_1 & -x_2 & x_3 & -x_1 - x_2 \\
x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & x_3^* \\
x_3 & x_4 & x_3 & -x_4 & x_3^* & x_4^* & x_3^* & x_4^* \\
x_4 & x_3 & -x_2 & x_1 & x_4 & -x_3 & x_2 & x_1 \\
\end{bmatrix} \\
X^{3/4}_{\text{complex}} &= \begin{bmatrix}
-x_2 & \frac{x_1}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & \frac{x_4}{\sqrt{2}} \\
x_2 & x_1 & x_4 & -x_3 & x_2^* & x_1^* & x_4^* & x_3^* \\
x_3 & x_4 & x_3 & -x_4 & x_3^* & x_4^* & x_3^* & x_4^* \\
x_4 & x_3 & -x_2 & x_1 & x_4 & -x_3 & x_2 & x_1 \\
\end{bmatrix} \\
\end{align*}
\]

Expressing terms of Equation (26) symbolically as follows
\[
\begin{align*}
\mathbf{y}_\text{eff} &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{bmatrix} \quad \text{and} \quad \mathbf{z}_\text{eff} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \end{bmatrix} \\
\end{align*}
\]

Using the orthogonality of the above effective channel matrix, the received signal is modified as
\[
\begin{align*}
\mathbf{y}_\text{eff} &= \mathbf{H}_\text{eff}^{\dagger} \mathbf{y}_\text{eff} \\
&= \frac{E_0}{\sqrt{3N_0}} \mathbf{H}_\text{eff}^{\dagger} \mathbf{H}_\text{eff} \mathbf{x}_\text{eff} + \mathbf{H}_\text{eff}^{\dagger} \mathbf{z}_\text{eff} \\
&= 2\frac{E_0}{\sqrt{3N_0}} \sum_{j=1}^{J} |h_j|^2 \mathbf{I}_4 \mathbf{x}_\text{eff} + \mathbf{z}_\text{eff} \\
\end{align*}
\]

Using the above result, the ML signal detection is performed as
\[
\hat{x}_{\text{ML}} = \arg \max \left( \frac{\mathbf{y}_\text{eff}^{\dagger} }{\sqrt{2\sum_{j=1}^{J} |h_j|^2}} \right) \quad i = 1, 2, 3, 4, \quad (28)
\]

We now consider the high rate space-time block code \( X^{3/4}_{\text{complex}} \) in Equation (22). Then we express the received signals as
\[
\begin{align*}
\mathbf{y} &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \\
\end{align*}
\]

Where, \( M_1 \) and \( M_2 \) are
\[
\begin{align*}
\mathbf{y} &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \\
\end{align*}

dsatisfies the constraint
\[
\begin{align*}
\end{align*}
\]
From equation (30), we can derive following three equations:

\[
\begin{bmatrix}
 y_1^* \\
 y_2^* \\
 y_3^* \\
 y_4^*
\end{bmatrix} = \sqrt{\frac{e_x}{3N_0}} \begin{bmatrix}
 h_1 & h_2 & h_3 & 0 & 0 & 0 \\
 h_2 & h_3 & h_2 & h_3 & h_3 & 0 \\
 h_3 & h_3 & h_2 & h_2 & h_3 & -h_1 \\
 -h_1 & -h_1 & h_3 & h_3 & h_3 & h_3
\end{bmatrix} \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 x_4
\end{bmatrix} + \begin{bmatrix}
 z_1 \\
 z_2 \\
 z_3 \\
 z_4
\end{bmatrix}
\]

(30)

Expressing terms of Equation (31) symbolically as follows

\[
\begin{bmatrix}
 y_1^1 \\
 y_2^1 \\
 y_3^1 \\
 y_4^1
\end{bmatrix} = \sqrt{\frac{e_x}{3N_0}} \begin{bmatrix}
 h_1 & h_2 & h_3 & 0 & 0 & 0 \\
 h_2 & h_3 & h_2 & h_3 & h_3 & 0 \\
 h_3 & h_3 & h_2 & h_2 & h_3 & -h_1 \\
 -h_1 & -h_1 & h_3 & h_3 & h_3 & h_3
\end{bmatrix} \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 x_4
\end{bmatrix} + \begin{bmatrix}
 z_1^1 \\
 z_2^1 \\
 z_3^1 \\
 z_4^1
\end{bmatrix}
\]

(31)

Expressing terms of Equation (32) symbolically as follows

\[
\begin{bmatrix}
 y_1^2 \\
 y_2^2 \\
 y_3^2 \\
 y_4^2
\end{bmatrix} = \sqrt{\frac{e_x}{3N_0}} \begin{bmatrix}
 h_1 & h_2 & h_3 & 0 & 0 & 0 \\
 h_2 & h_3 & h_2 & h_3 & h_3 & 0 \\
 h_3 & h_3 & h_2 & h_2 & h_3 & -h_1 \\
 -h_1 & -h_1 & h_3 & h_3 & h_3 & h_3
\end{bmatrix} \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 x_4
\end{bmatrix} + \begin{bmatrix}
 z_1^2 \\
 z_2^2 \\
 z_3^2 \\
 z_4^2
\end{bmatrix}
\]

(32)

From Equation (31), (32) and (33), the following decision statistics are derived:

\[
\begin{align*}
\tilde{y}_{eff,1} &= (h_{eff}^1)^H y_{eff}^1 = \sqrt{\frac{e_x}{3N_0}} (|h_1|^2 + |h_2|^2 + |h_3|^2) \\
\tilde{y}_{eff,2} &= (h_{eff}^2)^H y_{eff}^2 = \sqrt{\frac{e_x}{3N_0}} (|h_1|^2 + |h_2|^2 + |h_3|^2) \\
\tilde{y}_{eff,3} &= (h_{eff}^3)^H y_{eff}^3 = \sqrt{\frac{e_x}{3N_0}} (|h_1|^2 + |h_2|^2 + |h_3|^2)
\end{align*}
\]

(34)

(35)

(36)
Where $\mathbf{h}_{\text{eff},i}$ is the $i^{th}$ column of $\mathbf{H}_{\text{eff}}$, $i = 1, 2, 3$. Using the above results in eqn(34), eqn(35) and eqn(36), ML signal detection is performed as

$$\hat{r}_{i,\text{ML}} = Q\left( -\frac{2}{\pi} \frac{y_{\text{eff},i}}{\sqrt{2N_0 |h_i|^2}} \right), \quad i = 1, 2, 3. \quad (37)$$

Although construction of the effective channel for $X_{34}$, complex is rather more complex than the previous channel constructions [8], the detection processes in equations (34), (35) and (36) still have linear processing structures.

IV. SIMULATIONS

Simulations are done in MATLAB using the Rayleigh channel model described in Section II. We simulated $X$, $X_{34}$, complex, $X_{34}$, complex and $X_{34}$, complex for the case of $N_r = 1$ up to $N_r = 4$ while modulating using BPSK, QPSK, 4-QAM, 16-QAM, and 64-QAM gray mapping constellations. For each sample, blocks of 1000 symbols are simulated or until 100 bits in errors are obtained. The simulation is stopped when the $E_b/N_0$ (dB) progressively reaches 18dB in steps. Consequently, we obtained the bit error rate (BER) with each value of $E_b/N_0$ (dB).

V. RESULTS AND ANALYSIS

Keeping all other variables the same, the results obtained for BPSK and QPSK are nearly identical, and we therefore present data for QPSK and omit that of BPSK. Since the data is nearly identical, the reader can safely assume that the performance of BPSK is that of QPSK. We analyses the performance of each block code discussed earlier for the different cases of constant $N_r$ and coding rate.

---

For the case of $N_r$ constant, we fix $N_r = 1$. The result is shown in Figure 4. As expected, for each different code blocks, the performance degrades as more bits per symbol are transmitted. It can be observed that for a particular modulation and high SNR, the best performance is obtained by $X_{34}$, complex followed by $X_{34}$, complex, $X_{34}$, complex and $X$. However, for any modulation and low SNR, $X_{34}$, complex outperforms $X_{34}$, complex even when $X_{34}$, complex has greater coding gain. The results is that the best performance at low SNR is obtained by $X_{34}$, complex followed by $X_{34}$, complex, $X_{34}$, complex, $X_{34}$, complex, and $X$.

![Fig. 4 BER performance for OSTBCs with different code rates & modulation schemes using 1 Receive Antennas](image)

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Figure 5 shows the case where $N_r$ is fixed to 4. As can be observed, for a particular modulation, the best performance is obtained by $X_{34}$, complex followed by $X_{34}$, complex, $X_{34}$, complex, $X_{34}$, complex, and $X$. This order is the same as for the case of $N_r = 1$ and low SNR where $X_{34}$, complex, outperforms $X_{34}$, complex even though $X_{34}$, complex is having higher gain. One possible reason for this behaviour is that at higher rate $X_{34}$, complex causes lower channel gain per symbol and therefore higher BER for a particular SNR.

![Fig. 5 BER for OSTBCs with different code rates & modulation schemes using 4 Receive Antennas](image)
The BER curve for the case of keeping \( N_t = 4 \) constant while varying \( N_r \) from 1 to 4 for different modulations are depicted in Figure 6. It can be observed that for most of the modulation and block code, the gain of using 3 more antennas are approximately 12dB. However, between \( N_r = 1 \) and \( N_r = 2 \) the gain is approximately 7dB, between \( N_r = 2 \) and \( N_r = 3 \) the gain is approximately 3dB, and between \( N_r = 3 \) and \( N_r = 4 \) the gain is approximately 1.9dB. This result suggests diminishing returns as \( N_r \) increases. Another observation is that for any \( N_r \) and modulation scheme, \( X_{3,\text{complex}}^{1/2} \) and \( X_{4,\text{complex}}^{1/2} \) have a 3dB gain over \( X_{3,\text{complex}}^{3/4} \) and \( X_{4,\text{complex}}^{3/4} \) respectively. An interesting observation is that the performance of \( X_{4,\text{complex}}^{1/2} \) with \( N_r = 2 \) is similar to that of \( X_{4,\text{complex}}^{3/4} \) with \( N_r = 3 \), while \( X_{4,\text{complex}}^{1/2} \) with \( N_r = 1 \) is outperformed by \( X_{4,\text{complex}}^{3/4} \) with \( N_r = 2 \), and \( X_{4,\text{complex}}^{1/2} \) with \( N_r = 3 \) outperforms \( X_{4,\text{complex}}^{3/4} \) with \( N_r = 4 \).

VI. CONCLUSION

This paper provided a basic overview of MIMO systems. We briefly discussed MIMO channel modelling techniques. A basic introduction to Space-Time Coding was provided by presenting Alamouti’s scheme. We then discussed block codes schemes with different code rates for the cases of 3 and 4 transmit antennas. The encoding and decoding algorithms for each were presented. Simulation results were then presented. It was observed that higher diversity gain does not always imply better performance. This was observed when \( X_{4,\text{complex}}^{1/2} \) outperformed \( X_{4,\text{complex}}^{3/4} \) at low SNR for \( N_r = 1 \) and at any SNR for \( N_r = 2 \) up to \( N_r = 4 \). Similarly, it was observed that equal diversity gain does not imply equal performance. This was particularly demonstrated when \( X_{3,\text{complex}}^{1/2} \) outperformed all others for equal diversity gain. The penalty of having more transmit antennas, which consequently reduces the energy per transmit antenna was observed. Also, we observed diminishing returns for every scheme as the number of received antennas increased. It was particularly interesting to find that although \( X_{4,\text{complex}}^{3/4} \) and \( X_{4,\text{complex}}^{3/4} \) have higher rate than \( X_{3,\text{complex}}^{1/2} \) and \( X_{4,\text{complex}}^{1/2} \) the performance of \( X_{3,\text{complex}}^{1/2} \) and \( X_{4,\text{complex}}^{1/2} \) is greater and could therefore be preferred in some scenarios. Finally, we conclude that it is preferable to use a low constellation order with high code rate than high constellation order with low code rate.

References


Fig. 6 BER analysis for \( \frac{1}{2} \) rate code using 4Tx and 1, 2, 3 & 4Rx different modulation techniques.


