Extended Kalman Filter based State Estimation of Wind Turbine

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Abstract - State estimation provides the best possible approximation for the state of the system by processing the available information. In the proposed work, the state estimation technique is used for the state estimation of wind turbine. Modern wind turbines operate in a wide range of wind speeds. To enable wind turbine operation in such a variety of operating conditions, sophisticated control and estimation algorithms are needed. The theoretical basis of Extended Kalman Filter algorithm is explained in detail and performance is tested with the simulation.

A nonlinear state estimator named Extended Kalman Filter can be used for estimating the states of wind turbine. The Extended Kalman Filter is a recursive estimator that can be decomposed into two phases such as prediction and correction performed at every time instant. The states estimated by using Extended Kalman Filter for wind turbine application includes rotor speed of turbine, tower top displacement and its velocity.

**Keywords** – Extended Kalman Filter, Modelling, state estimation, wind turbine.

I. INTRODUCTION

Wind energy is one of the most popular types of renewable energy being explored. It has huge potential to fulfill our future power requirements. Wind turbine is a device that is used for converting kinetic energy from the wind into mechanical energy. Wind turbine has high nonlinearity and it is not possible to use a linear model for control design and state estimation. To this extent we introduce Extended Kalman Filter Algorithm. The Extended Kalman Filter (EKF) is based on the nonlinear wind turbine model that includes the rotor speed, tower top displacement and its velocity.

State estimation is the process of estimating the values of parameters based on measured data having random component. The parameters explain the underlying physical setting in such a way that the distribution of the measured data are affected by their values. An estimator is used to approximate the unknown parameters by using the values of measurement data. Many types of estimators are available. The commonly used estimator is the Kalman Filter and its types.

The Kalman Filter is also known as Linear Quadratic Estimation (LQE). It is an algorithm which uses measurements data taken over time, containing noise i.e. random variations and other inaccuracies. It produces estimates of unknown variables that are more precise than the single measurement alone. It operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state. The Extended Kalman Filter (EKF) is known as the nonlinear version of the Kalman filter which linearizes about an estimate of the current mean and covariance.

The format of paper presentations is as follows: In Section II, the mathematical model of wind turbine is given. In Section III, the nonlinear state estimation by using EKF is introduced. In Section IV, simulation implementations are evaluated for the performances of the EKF. Finally, the concluding remarks are stated in Section V.

II. MATHEMATICAL MODELLING OF WIND TURBINE

Very common method used for wind turbine modelling is blade element and momentum theory which yields reliable and detailed wind turbine model. However, such models are implicit equations which are not suitable for controller design. Therefore simple model uses quasi steady state relations for controller design. Fig.1 shows a general wind turbine.
The motion of the rotor can be described with the equation

\[ J \ddot{\omega} = M_a - M_g \]  
(1)

where \( \omega \) is rotor speed, \( M_g \) is generator electromagnetic torque, \( M_a \) is aerodynamic torque and \( I_t \) is turbine moment of inertia.

The aerodynamic torque can be computed as

\[ M_a = \frac{\pi}{2} \rho_a R_b^2 V_c^2 C_Q(\nu_o, \omega, \beta) \]  
(2)

where \( V_c \) is effective wind speed on wind turbine rotor, \( R_b \) is blade length and \( \rho_a \) is air density and \( C_Q \) is torque coefficient.

Important parts of wind turbine dynamics are tower oscillations. The first harmonics in tower oscillations is the most dominant, so tower dynamics can be approximated by

\[ M \ddot{x}_t + D \dot{x}_t + C x_t = F_t \]  
(3)

where \( x_t \) is the tower top deflection, \( M, D \) and \( C \) are modal mass, damping and stiffness respectively.

The aerodynamic thrust force is defined similarly as

\[ F_t = \frac{\pi}{2} \rho_a R_b^3 \nu_o^2 C_T(\nu_o, \omega, \beta) \]  
(4)

where thrust coefficient \( C_T \) describes the steady state dependence of aerodynamic thrust force on wind speed, rotor speed and pitch angle.

In order to estimate states of wind turbine such as rotor speed, tower top displacement and its velocity, a state space model is needed. Nonlinear mathematical model can now be written as space form as follows:

\[ \dot{x} = Ax + Bu \]  
(5)

with matrices \( A \) and \( B \)

\[
A = \begin{bmatrix}
M_o & 0 & -M_v \\
J_t & 0 & -F_v \\
F_m & C & F_v + D
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
M_o & 0 & -M_v \\
J_t & 0 & -F_v \\
F_m & C & F_v + D
\end{bmatrix}
\]

where \( M, D, C, J_t \) are modal mass, damping, stiffness and turbine moment of inertia.

\[ M_o, M_v, F_o, F_v, F_m \] are derivatives of aerodynamic torque and thrust force. The state vector and input can be given as

\[
x = \begin{bmatrix}
\omega \\
x_t \\
\dot{x}_t \\
\nu_o \\
\beta \\
M_e
\end{bmatrix}
\]

The output is given by \( y(k) \)

\[ y(k) = C x(k) \]

where \( C = \begin{bmatrix} 1 & 0 \end{bmatrix} \)

These equations are used by Extended Kalman Filter to estimate the states of wind turbine.
### Table 1: Parameters of Wind Turbine

<table>
<thead>
<tr>
<th>S.No</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( M_\omega )</td>
<td>0.08266 Nms/rad</td>
</tr>
<tr>
<td>2.</td>
<td>( M_V )</td>
<td>1.26908 Ns</td>
</tr>
<tr>
<td>3.</td>
<td>( F_\omega )</td>
<td>19.774 Ns/rad</td>
</tr>
<tr>
<td>4.</td>
<td>( F_V )</td>
<td>5.0641 Ns/m</td>
</tr>
<tr>
<td>5.</td>
<td>( F_B )</td>
<td>19.774 Ns/rad</td>
</tr>
<tr>
<td>6.</td>
<td>( J_t )</td>
<td>4 Nms(^2)</td>
</tr>
<tr>
<td>7.</td>
<td>( M )</td>
<td>2.321 Ns/m</td>
</tr>
<tr>
<td>8.</td>
<td>( D )</td>
<td>4.672 Ns/m</td>
</tr>
<tr>
<td>9.</td>
<td>( C )</td>
<td>8.464 N/m</td>
</tr>
</tbody>
</table>

Finally, a discrete model of the wind turbine is represented as

\[
x(k+1) = \Phi(x) + \Gamma u(k)
\]

which can be obtained from continuous time model with following approximation (Franklin et al. 1997).

\[
\Phi = e^{AT} \approx I + AT
\]

\[
\Gamma = \int_0^T e^{AT}Bdt \approx BT
\]

where \( T \) is sampling time.

### III. EKF Algorithm

Extended Kalman Filter (EKF) is known as the nonlinear version of the Kalman Filter which linearizes about an estimate of the current mean and covariance. In the Extended Kalman Filter, the state transition and observation models need not be linear functions of the state but may instead be differentiable functions.

\[
x_k = f(x_{k-1}, u_{k-1}) + w_{k-1}
\]

Finally, a discrete model of the wind turbine is represented as

\[
z_k = h(x_k) + v_k
\]

where \( w_k \) and \( v_k \) are the process and observation noises which are both assumed to be zero mean multivariate Gaussian noises with covariance \( Q_k \) and \( R_k \) respectively. The function \( f \) can be used to compute the predicted state from the previous estimate and similarly the function \( h \) can be used to compute the predicted measurement from the predicted state. However, \( f \) and \( h \) cannot be applied to the covariance directly. Instead a matrix of partial derivatives is computed.

At each time step, the Jacobian is evaluated with current predicted states. These matrices can be used in the Kalman Filter equations. This process essentially linearizes the nonlinear function around the current estimate.

\[
A_k = f'(\hat{x}_k, u_k)
\]

\[
C_k = h'(\hat{x}_k)
\]

The derivatives are taken with respect to \( x \) and then evaluated at \( \hat{x}_k = \hat{x}_k \).

The Kalman gain and the error covariance matrix are computed by the execution of following steps.

\[
K_k = P_kC_k^T(C_kP_kC_k^T + R)^{-1}
\]

\[
\hat{x}_{k+1} = f'(\hat{x}_k, u_k) + K_k(y_k - h(\hat{x}_k))
\]

\[
P_{k+1} = A_k(I-K_kC_k)P_kA_k^T + Q
\]

The above steps are repeated to estimate the states of a nonlinear system. The state space representation of the system is as follows

\[
\frac{dX}{dt} = AX + BU
\]

\[
Y = CX
\]

For digital implementation of the EKF, the discretized equations are required. These can be obtained from the state space representation of the system.
\[ X(k + 1) = A_d X(k) + B_d U(k) \]  
(18)

\[ Y(k) = CX(k) \]  
(19)

\[ A_d = I + AT \]  
(20)

\[ B_d = BT \]  
(21)

The \( T \) represents the sampling time. To achieve adequate accuracy, the sampling time should be smaller than the characteristic time constants of the turbine. The final choice for the sampling time should be based on obtaining adequate execution time of the full EKF algorithm and also satisfactory accuracy and stability. The system noise is represented by \( v(k) \) (\( v \)-noise vector of the states) which is assumed to be zero-mean and white Gaussian noise. It is independent of \( X(k) \) and its covariance matrix is \( Q \) and the system model becomes

\[ X(k + 1) = A_d X(k) + B_d U(k) + v(k) \]  
(22)

The measurement noise is represented by \( w(k) \) which is assumed to be zero-mean and white Gaussian noise. It is independent of \( X(k) \) and \( V(k) \) and its covariance matrix is \( P \) and the output equation becomes

\[ Y(k) = CX(k) + w(k) \]  
(23)

The purpose of the Kalman filter is to obtain the unmeasurable states by using the measured states and also the statistics of the noise and measurements. In general, the computational inaccuracies, modelling errors and errors in the measurements are considered by means of noise inputs. A critical part of the design is to use correct initial values for the covariance matrices. The elements of \( Q \) and \( R \) depend on the number of state variables. The system noise matrix \( Q \) is a three by three matrix and the measurement noise matrix \( R \) is a three by three matrix.

**IV. SIMULATION RESULTS**

Extended Kalman Filter provides the estimates of wind turbine such as rotor speed, tower top displacement and its velocity. The estimated outputs for wind turbine are shown in Fig 2,3,4.
Fig. 4. Plot of estimated and true value of tower top velocity

From the above estimated plots it is observed that Extended Kalman Filter provides best estimates of wind turbine, highly non linear system.

V. CONCLUSIONS

The application of wind turbine is very essential nowadays and the state estimation provides to have better understanding of the behaviour of the system. Rotor speed, tower top displacement and its velocity are estimated using Extended Kalman Filter algorithm and it is shown that EKF provides best accuracy for highly non linear systems. It is possible to implement this estimation method for all other wind turbine states such as position of each blade tip, pitch velocity, velocity of each blade tip along blade coordinate system and so on. During weak winds, the control system has to optimize wind energy conversion by using appropriate generator torque and during strong winds, wind turbine has to be constrained. An efficient way to constrain wind energy capture is obtained by pitch control which is the future scope of this project.

REFERENCES


