Authentication of Algebraic Properties of Optimistic and Pessimistic Multi-Granular Rough Sets

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Abstract—Rough set theory was introduced by Pawlak [1, 2] mainly to handle uncertainty and impreciseness. The rough set theory got extended to multi-granular based rough sets by Qian, Y.H and Liang, J.Y [3, 4]. In multi-granular rough sets, the granules were represented through multi-equivalence relations. Since it uses multi-equivalence relations it was highly enriched to handle practical applications. In general, the algebraic property used to deal with elements and studies the relationship among the elements. In this paper, the database based validation for algebraic property of multigranular rough sets particularly associative, commutative properties are provided. Also this paper contains the database based validation for the conditions under which the two types of multigranulations reduced to single granulation.

Index Terms— algebraic property, multi-granular rough sets, equivalence relations, database

I. INTRODUCTION
Algebraic properties generally deal with elements and study of the relationship of elements. Lower approximations of rough sets and upper approximations of rough sets are having their similarity with mathematical morphology in the operators used. Lower approximation involve subset operation as like erosion and opening, while upper approximations involve set intersection as do dilation and closing [7]. Applying algebraic erosion, dilation, opening and closing for defining the corresponding approximations of rough sets lead to the concept called algebraic rough sets [7]. Hence the morphological operators seem to be a good tool for defining lower and upper approximation of rough sets. Moreover these operators lead to a generalization of rough sets. In this context the concept of algebraic properties of general rough sets got extended in view with multigranulation rough sets. In general rough set theory some of the interesting properties were studied and obtained on intersection and union of rough sets of different types [5]. This study has been further extended to multigranulation rough sets to find out the validity of other algebraic properties like associativity and commutativity [6].

II. ALGEBRAIC PROPERTIES OF MULTIGRANULATIONS
With the same notations as in previous Theorem, the following algebraic associative and commutative properties are satisfied by ‘+’ and ‘*’ type of multigranulation rough sets.

\[
R + SX = S + RX \quad \text{and} \quad \overline{R + SX} = \overline{S + RX}
\]
\[
(R + S) + TX = R + (S + T)X \quad \text{and} \quad \overline{R + S} + TX = \overline{R + (S + T)X}
\]
\[
R * SX = S * RX \quad \text{and} \quad \overline{R * SX} = \overline{S * RX}
\]
\[
(R * S) * TX = R * (S * T)X \quad \text{and} \quad \overline{(R * S) * TX} = \overline{R * (S * T)X}
\]
III. VALIDATION ON ASSOCIATIVE AND COMMUTATIVE PROPERTY ON OPTIMISTIC MULTI-GRAINULAR ROUGH SETS

The following database is used to validate the associative and commutative property of optimistic multigranulation rough sets.

<table>
<thead>
<tr>
<th>Faculty Name</th>
<th>Division</th>
<th>Grade</th>
<th>Highest Degree</th>
<th>Native State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sam</td>
<td>NW</td>
<td>AP</td>
<td>M.C.A</td>
<td>Tamil Nadu</td>
</tr>
<tr>
<td>Ram</td>
<td>IS</td>
<td>Pr</td>
<td>Ph.D</td>
<td>Andhra Pradesh</td>
</tr>
<tr>
<td>Shyam</td>
<td>SE</td>
<td>APJ</td>
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<td>Tamil Nadu</td>
</tr>
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<td>AP</td>
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<tr>
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<td>ASP</td>
<td>Ph.D</td>
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<tr>
<td>Jacob</td>
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<tr>
<td>Lakman</td>
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<table>
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<td>Preetha</td>
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<td>SP</td>
<td>Ph.D</td>
<td>Karnataka</td>
</tr>
</tbody>
</table>

The database which is stated above is used for the validation. Here NW indicates networks, SE indicates Software engineering, AI indicates Artificial Intelligence, ES indicated Embedded Systems, IS indicated Information Systems. Similarly AP indicates Assistant Professor, APJ indicates Assistant Professor (Junior), ASP indicates Associate Professor, SP indicates Senior Professor, Pr indicates Professor.

The universe and other equivalence relations are defined below.

\[ U = \{\text{shyam, albert, mishra, jacob, sam, john, sita, fatima, ram, peter, roger, hari, smith, keny, linz, williams, mukherjee, preetha}\} \]

\[ U/\text{Highest Degree} = \{\{\text{shyam, albert, mishra, jacob}\}, \{\text{sam, john}\}, \{\text{sita, fatima}\}, \{\text{ram, peter, roger, hari, smith, keny, linz, williams, mukherjee, preetha}\}\} \]

\[ U/\text{Native State} = \{\{\text{sam, shyam, roger, mishra, williams}\}, \{\text{ram, peter}\}, \{\text{hari, smith, linz}\}, \{\text{peter, john, lakman, fatima, mukherjee}\}, \{\text{keny, martin, jacob, sita, pretha}\}\} \]

\[ U/\text{Division} = \{\{\text{sam, smith, jacob}\}, \{\text{shyam, john, keny, lakman, pretha}\}, \{\text{peter, albert, linz, sita}\}, \{\text{roger, mishra, williams, fatima}\}, \{\text{ram, hari, martin, mukherjee}\}\} \]

Let

\[ U/P \text{ be highset degree} \]
\[ U/Q \text{ be Native state} \]
\[ U/R \text{ be Division} \]
\[ X = \{\text{sam, john}\} \]
\[ P + QX = \{\text{sam, john}\} \]
\[ Q + PX = \{\text{sam, john}\} \]
Hence validated.

To Prove \( P + QX = Q + PX \)
Let \( X = \{\text{sam, john}\} \)
\[ P + QX = \sim (P + Q(\sim X)) = \sim ([x]_p \subseteq X \lor [x]_q \subseteq X) = \sim (\text{shyam, albert, mishra jacob, sita, fatima, ram, peter, roger, hari, smith, keny, linz, williams, mukherjee, preetha}) = \{\text{sam, john}\} \]
\[ Q + PX = \sim (Q + P(\sim X)) = \sim ([x]_p \subseteq X \lor [x]_q \subseteq X) = \sim (\text{shyam, albert, mishra jacob, sita, fatima, ram, peter, roger, hari, smith, keny, linz, williams, mukherjee, preetha}) = \{\text{sam, john}\} \]
Hence validated.

To Prove \( P + Q + RX = P + (Q + R)X \)
\[ X = \{\text{sam, john, ram}\} \]
\[ P + Q + RX = ([x]_p \subseteq X \lor [x]_q \subseteq X) \lor [x]_r \subseteq X = (\{\text{sam, john}\} \lor \phi) \lor \phi = \{\text{sam, john}\} \]
\[ P + (Q + R)X = [x]_p \subseteq X \lor ([x]_q \subseteq X \lor [x]_r \subseteq X) \lor = (\{\text{sam, john}\} \lor \phi) \lor \phi = \{\text{sam, john}\} \]
so \( P + Q + RX = P + (Q + R)X \) Hence Validated.

To Prove \( P^*QX = Q^*PX \)
Let \( X = \{\text{sam, ram, shyam, peter, roger, albert, mishra, hari, smith, linz, keny, williams, martin, jacob, lakman, sita, mukherjee, preetha}\} \)
\[ P^*QX = \sim (P^*Q(\sim X)) = \sim ([x]_p \subseteq X \lor [x]_q \subseteq X) = \sim (\{\text{sam, john, sita, fatima}\} \lor \{\text{hari, smith, peter, john, lakman, fatima, mukherjee}\}) = \sim (\{\text{john, fatima}\}) = \{\text{sam, ram, shyam, peter, roger, albert, mishra, hari, smith, linz, keny, williams, martin, jacob, lakman, sita, mukherjee, preetha}\} \]
\[ Q^*PX = \sim (Q^*P(\sim X)) = \sim ([x]_p \subseteq X \lor [x]_q \subseteq X) = \sim (\{\text{john, lakman, fatima, mukherjee}\} \lor \{\text{sam, john, sita, fatima}\}) = \sim (\{\text{john, fatima}\}) = \{\text{sam, ram, shyam, peter, roger, albert, mishra, hari, smith, linz, keny, williams, martin, jacob, lakman, sita, mukherjee, preetha}\} \]
so \( Q^*PX = P^*QX \) Hence Validated.

To Prove \( P^*QX = Q^*PX \)
\[ X = \{\text{sam, ram, shyam, peter, roger, albert, mishra, hari, smith, linz, keny, williams, martin, jacob, lakman, sita, mukherjee, preetha}\} \]
\[ P^*QX = [x]_p \subseteq X \lor [x]_q \subseteq X = \{\text{shyam, albert, mishra, martin, jacob}\} \]
\[ Q^*PX = [x]_p \subseteq X \lor [x]_q \subseteq X = \{\text{shyam, albert, mishra, martin, jacob}\} \]
So \( P^*QX = Q^*PX \) Hence Validated.

To Prove \( (P^*Q)*RX = P^*(Q*R)X \)
\[ X = \{\text{shyam, albert, mishra, martin, jacob, ram, linz, peter, sita}\} \]
\[ (P^*Q)*RX = ([x]_p \subseteq X \lor [x]_q \subseteq X) \lor [x]_r \subseteq X = ([\text{shyam, albert, mishra, martin, jacob}] \lor \{\text{ram, albert}\}) \lor \{\text{ram, albert}\} \]
\[ = \{\text{albert}\} \]

IV. AUTHENTICATION ON ASSOCIATIVE AND COMMUTATIVE PROPERTY ON PESSIMISTIC MULTI-GRAΝULATION BASED ROUGH SETS
P*(Q*R)X = \{x_p \subseteq X \text{ and } (\{x_q \subseteq X \text{ and } \{x_r \subseteq X\})
= \{\text{shyam, albert, mishra, martin, jacob}\} \text{ and } \{\text{ram, albert}\}
= \{\text{shyam, albert, mishra, martin, jacob}\} \text{ and } \{\text{albert}\}

So P*(Q*R)X = (P*Q)*RX \quad \text{Hence validated}

To Prove \((P*Q)*RX = P*(Q*R)X\)

\((P*Q)*RX = \sim ((P*Q)*R(\sim X))\)

\(\sim ((\{x_p \subseteq X \text{ and } \{x_q \subseteq X \text{ and } \{x_r \subseteq X\})\))\)

\(\sim ((\{\{\text{sam, john, sita, fatima}\} \text{ and } \{\text{hari, smith, linz, peter, john, lakman, fatima, mukherjee}\}) \text{ and } \{\phi\}))\)

\(\sim (\{\{\phi\}) = \sim (\phi) = \bigcup\)

\(P*(Q*R)X = \sim (P*(Q*R)(\sim X))\)

\(\sim ((\{x_p \subseteq X \text{ and } (\{x_q \sim X \text{ and } \{x_r \subseteq X\})\}))\)

\(\sim ((\{\{\text{sam, john, sita, fatima}\} \text{ and } (\{\text{hari, smith, linz, peter, john, lakman, fatima, mukherjee}\} \text{ and } \{\phi\}))\)

\(\sim (\{\{\text{sam, john, sita, fatima}\} \text{ and } \{\phi\}) = \sim (\phi) = \bigcup\)

so \(P*(Q*R)X = (P*Q)*RX \quad \text{Hence validated}\)

V. CONCLUSION

This paper provided database based validation and authentication on associative and commutative property on optimistic and pessimistic multigranulation rough sets. More particularly here the associative property is used to prove how even different grouping of equivalence relations with respect to similar approximations gives the same result. In future work, this would be extended to suitable industrial applications where such grouping of different phases of product manufacturing gives no change in performance evaluation of a developed product.

VI. REFERENCES


VII. BIOGRAPHY

R. Raghavan is an Assistant Professor (Senior) in the School of Information Technology and Engineering (SITE), VIT University at Vellore in India. He obtained his masters in computer applications from University of Madras. He completed his M.S., (By Research) in information technology from School of Information Technology and Engineering (SITE), VIT University. He is a life member of CSI. His current research interest includes Rough Sets and Systems, Knowledge Engineering, Granular Computing, Intelligent Systems, image processing.