Application of Least Squares Parameter Estimation Techniques in Fault Detection

Absal Nabi

Assistant Professor, Electrical and Electronics Department, Ilahia College of Engineering and Technology,

Muvattupuzha, Kerala, India

***Abstract*—Failure detection has been the subject of many studies in the past. Modern technology has required highly complex dynamic systems. A critical system is any system whose 'failure' could threaten the system's environment or the existence of the organization which operates the system. A fault is understood as any kind of malfunction in the actual dynamic system, the plant that leads to an unacceptable anomaly in the overall system performance. Fault detection via parameter estimation relies in the principle that possible faults in the monitored system can be associated with specific parameters and states of the mathematical model of the system given in the form of an input-output relation. In this thesis, the focus is put on the study of fast least squares parameter estimation methods, like recursive least square (RLS) algorithm, Fast Kalman algorithm, FAEST(fast a priori error sequential technique) algorithm, FTF(fast transversal filter) algorithm and lattice filter algorithm and their fast algorithm implementation. The above algorithms are applied to a dynamic system and the performances of different algorithms in detecting different changes in the systems are compared. The MATLAB coding of these algorithms are done and their effect on first and second order dynamic systems under various conditions are verified. Statistical methods like Shewart moving range control chart, the cumulative sum control chart, the moving average control chart, the exponentially weighted moving average control chart (EWMA) etc. for analyzing the changes in dynamic systems are also studied.**

Keywords: Fault Detection, Parameter Estimation, Recursive Least Squares, Fast Least Squares, Statistical Control Chart

I. INTRODUCTION

An A fault is understood as any kind of malfunction in the actual dynamic system. Failure detection has been the subject of many studies in the past. Modern technology has required highly complex dynamical systems. A critical system is any system whose 'failure' could threaten the system's environment or the existence of the organization which operates the system. Considering the increased structural and operational complexity of safety critical systems, some consequences of fault can be extremely serious. For safety critical systems an important means of increase in dependability is to detect and identify the different types of failure, furthermore, to accommodate or minimize the impact of failures. It is desirable to detect and identify the different types of failures that occurred in the system for the stability and performance of the system. The primary objective of fault detection is to detect and identify the actuator, sensor and component failures, preventing the system from getting into undesirable state. Fault detection via parameter estimation relies in the principle that possible faults in the monitored system can be associated with specific parameters and states of the mathematical model of the system given in the form of an input-output relation.

II. THE LEAST SQUARES METHOD

The least square method, a very popular technique is used to compute estimations of parameters and to fit data. Least mean squares (LMS) algorithms are used in adaptive filters to find the filter coefficients that relate to producing the least mean squares of the error signal (difference between the desired and the actual signal). It is a stochastic gradient descent method in that the filter is only adapted based on the error at the current time. The idea behind LMS filters is to use the method of steepest descent to find a coefficient vector which minimizes a cost function. The cost function is defined as

$$C(n) = E\{|e(n)|^2\}$$

where $e(n)$ is the error signal and $E\{\,\}$ denotes the expected value.

III. THE RECURSIVE LEAST SQUARE ALGORITHM

Recursive least squares (RLS**)** algorithm is used in adaptive filters to find the filter coefficients that relate to recursively producing the least squares (minimum of the sum of the absolute squared) of the error signal (difference between the desired and the actual signal).The recursive least square (RLS) problem is an extension of the ordinary least square problem. The problem is simply stated as follows. At time k, we have observations $x(1), x(2), \ldots\ldots, x(k)$ and desired responses $d(1), d(2), \ldots\ldots, d(k)$.Assume that the weight vector which solves the LS problem for the available observations and desired responses has been computed. As we obtain a new measurement $x(k+1)$ and a new desired response $d(k+1)$,we would like to update the previous LS solution using the new data rather than recomputing the LS solution from scratch.

IV. FAST LEAST SQUARES ALGORITHMS

Due to the versatility as well as its ease of implementation, the fast least squares algorithms are attractive for many adaptive filtering applications. Fast recursive least squares algorithms such as the fast Kalman algorithm, the FAEST algorithm, and the FTF algorithm perform least squares adaptive filtering with low computational complexity, which is directly proportional to the filter length. The Fast Kalman algorithm primarily used a priori prediction errors for the adaptation gain computations. If a posteriori prediction errors are used, it is possible to reduce the number of multiplications even further. This is the idea behind the next FLS algorithm, which is known as the fast a priori error sequential technique (FAEST)

The FAEST algorithm is based on the a priori adaptation gain $t_N(k)$ rather than the a posteriori gain $g_N(k)$.We begin by computing the apriori forward and backward prediction errors. The fast transversal filters (FTF) algorithm is nearly identical in structure to the FAEST algorithm, with one important difference, the way in which the conversion factor $\varphi_N(k)$ is updated.

Recursive least squares can also be performed with a lattice structure. The lattice structures require the use of time and order recursions. The recursive least squares lattice algorithms require more computations than their transversal counterparts, but result in better numerical behaviour and generate adaptive filters of all intermediate orders, which is useful when the proper order is not known ahead of time.

V. COMPARISON OF COMPUTATIONAL COMPLEXITYOF DIFFERENT ALGORITHMS

RLS algorithm requires roughly N^2 multiplications per iteration. It can be seen that Fast Kalman algorithm requires roughly 8N multiplications for the adaptation gain and 2N multiplications for the filtering computation. This represents a substantial improvement over the N^2 multiplications required by direct application of the RLS algorithm.The computational complexity of the FAEST algorithm is roughly 6N multiplies for the adaptation gain updating and another 2N multiples for the filtering operation. This represents nearly a 20% reduction in complexity over the Fast Kalman algorithm. The recursive least squares lattice algorithms require more computations than their transversal counterparts, The computational complexity of the FTF algorithm is essentially identical to the FAEST algorithm: roughly 5N multiplies for the adaptation gain updating and 2N multiplies for the filtering operation.

VI. CHANGE DETECTION USING FAST LEAST SQUARES ALGORITHM

Now we can make use of the fast least squares algorithm for detecting different kinds of changes in dynamic systems. The systems considered here are first order and second order filters which are characterized by the equations

$$H(z) = \frac{\beta}{1-\propto z^{-1}} \; and$$

$$H(z) = \frac{\beta}{(1-\propto z^{-1})(1-\propto z^{-1})} \; respectively.$$

These systems are incorporated with different adaptive filters which make use of the different fast least squares algorithms. The input to the system is a sine wave added with some noise. Before considering the different cases the two systems are checked with the above algorithms by taking different values for the number of samples. After that the two systems are checked with the above algorithms by varying the amplitude value of the input signal. Then the two systems are checked with the above algorithms by simultaneously varying the amplitude value and number of samples.

For the different cases considered here, the number of samples is taken as 2000 and the filter length is taken as 10.

The different cases consider for first order system are

Case 1: for 1-1000 samples $\propto=\propto_0$

for 1001-2000 $\propto=\propto_1$

In this case the system is checked with different values of signal to noise ratio

Case 2: for 1-1000 $\propto=\propto_0$

for $1001-1100$ $\propto=\propto_0+\frac{\propto_1-\propto_0}{n_2-n_1}*n-n_1$ where $n_1 = 1001 \; and \; n_2 = 1100$

for 1101-1200 $\propto=\propto_1$

for 1201-2000 $\propto=\propto_0$

Case 3: for 1-1000 $\beta = \beta_1$

for 1001-1100 $\beta = \beta_2$

for 1101-2000 $\beta = \beta_1$

The different cases consider for second order system

Case 1: for 1-1000 samples $\propto=\propto_0$

for 1001-2000 $\propto=\propto_1$

In this case the system is checked with different values of signal to noise ratio

Case 2: for 1-1000 $\propto=\propto_0$

for 1001-1100 $\propto=\propto_1$

for 1101-1200 $\propto=\propto_2$

for 1201-2000 $\propto=\propto_0$

Case 3: for 1-1000 $\beta = \beta_1$

for 1001-1100 $\beta = \beta_2$

for 1101-2000 $\beta = \beta_1$

The adaptive filter coefficients are taken after every 10^{th} iteration and the graphs between filter coefficients and numbers of samples for different cases are plotted for different algorithms.

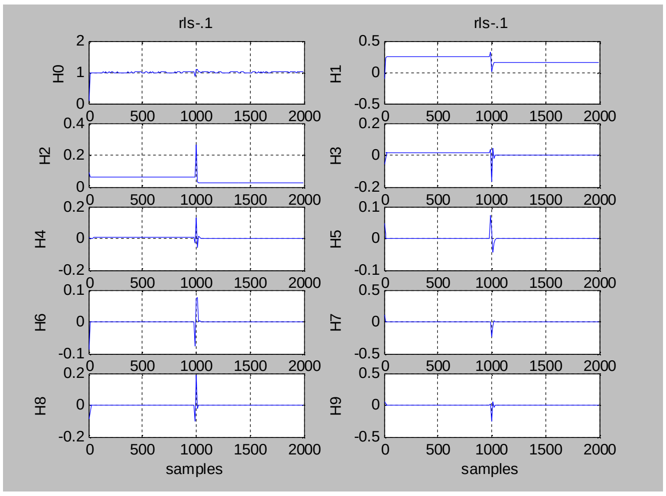

Fig. 1 Firsr Order RLS Case 1

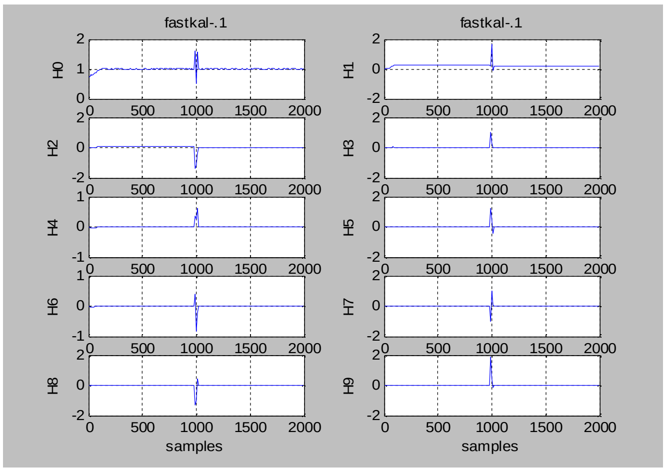

Fig. 2 First Order Fast Kalman Case 1

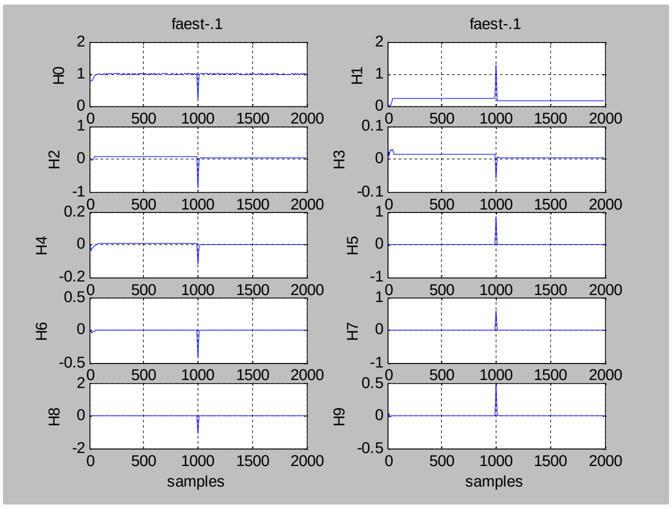

Fig. 3 First Order FAEST Case 1

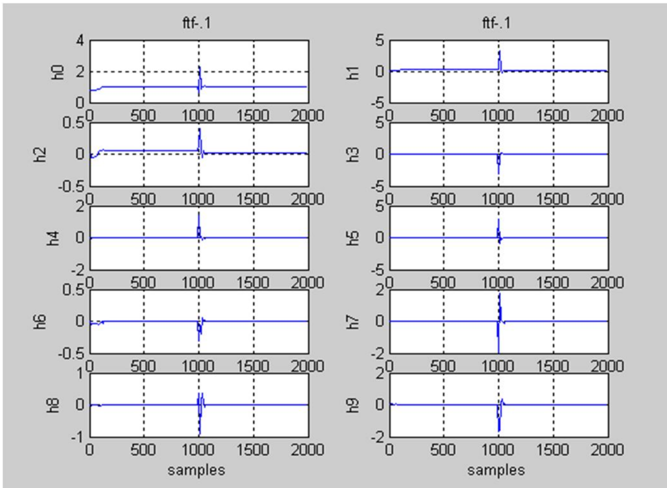

Fig. 4 First Order FTF case 1

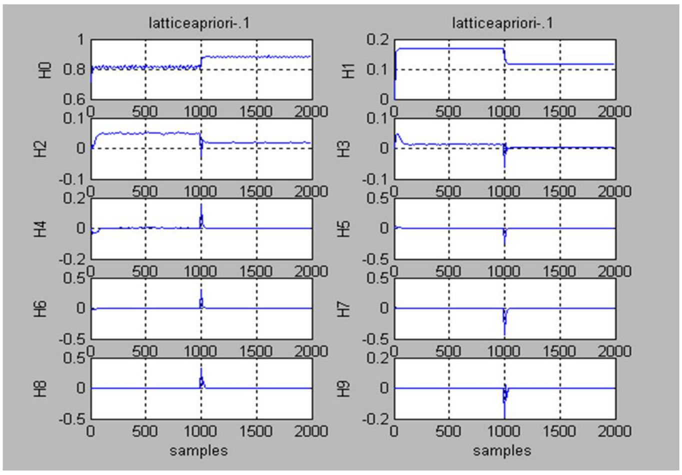

Fig. 5 First Order Lattice A Priori Case 1

VII. ANALYSIS OF THE CHANGES

For analyzing the changes in the dynamic systems detected by the fast least squares algorithms and to check whether we make use of some statistical methods. By using these statistical methods we check whether the changes are faults or not. Different control chart methods like the Shewhart Moving Range Control Chart, $\bar{X}$ and S charts, the Cumulative Sum (CUSUM) Control Chart, and the Exponentially Weighted Moving Average (EWMA) Control Chart are considered. But only the Shewhart Moving Range Control Chart method work well with the systems we consider. The performances of other systems are not satisfactory. So we proceed with The Shewhart Moving Range

Control Chart method. The Shewhart Moving Range Control Chart method is based on moving range of two successive observations. If xi and xi-1 are two successive measurements, then the moving range is defined as MRi = | xi- xi-1|.The lower control limit (LCL) is taken as 0 and the upper control limit (UCL) is taken as 4* Sum of the moving ranges.. .Here the moving range is calculated for each set of the adaptive filter coefficients. The MATLAB coding of The Shewhart Moving Range Control Chart method is done and is used in analyzing the changes in the earlier described systems detected using fast least squares algorithms.

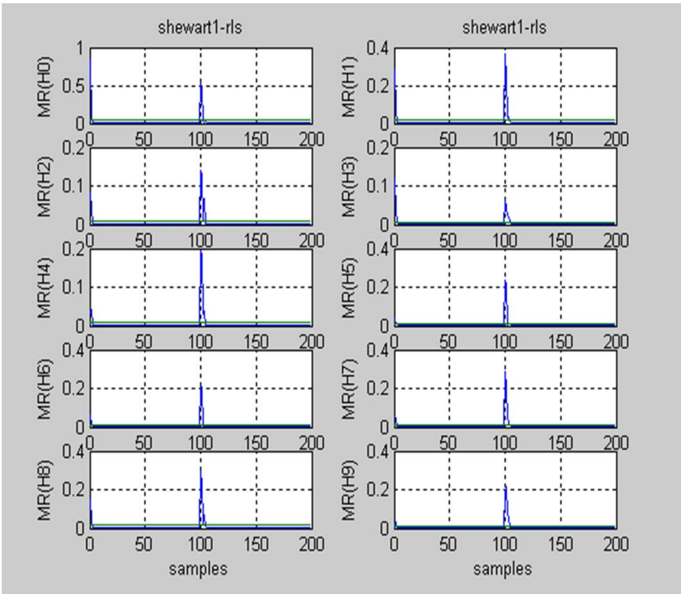

Fig. 6 Shewart Chart and RLS Algorithm

VIII. CONCLUSION

With today's readily available computing power, one could not help but to think and re-think some of the existing computationally more efficient, numerically less perfect algorithms. A new approach towards the change detection of linear dynamical systems using fast least squares parameter estimation techniques are proposed in this thesis. Here we consider only simple first order and second order systems. But the idea can be effectively extended to complex critical systems also. The results presented in this paper is significant in the sense that they have important role in signal processing, communication, control and other engineering applications

REFERENCES

[1] *George Vachtsevanos, Intelligent Fault Diagnosis and Prognosis For Engineering Ststems, First Edition, John Wiley&Sons, Inc, 2006*

[2] *Otto S., and Denier J.P., an Introduction to Programming And Numerical Methods in MATLAB, Low Price Edition, New Age International, 2006*

[3] *S.Haykin, Adaptive Filter Theory, Prentice-Hall, Second edition1986*

[4] *C.Hajivev, F.Caliskan, Fault Diagnosis and Reconfiguration in Flight Control Systems, First Edition, Springer Publishers, 2003*

[5] *Janos Gertler, Fault Detection and Diagnosis in Engineering Systems, Marcel Dekker Publishers, 1998*

[6] *Alexander Ilin, Harri Valpola, and Erkki Oja, Nonlinear Dynamica Factor Analysis for State Change Detection, IEEE Transactions on Neural Networks, Vol. 15, No. 3, pages 559-575, May 1997*